Decentralized Widely Linear Precoding Design for the MIMO Interference Channel

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Abstract—**This paper addresses the interference management in a MIMO interference channel (MIMO-IC) by proposing a decentralized transmit and receive beamformer optimization using improper (or circularly asymmetric complex) Gaussian signaling. For the ease of exposition, the downlink (DL) of a cellular network is considered. In order to generate improper Gaussian signals, widely linear precoding (WLP) is adopted at transmission, while at reception we consider that users might apply either widely linear estimation (WLE) or linear estimation. The coordination between transmitters for WLP design is attained by taking into account the received signal in the uplink (UL), provided that propagation channel reciprocity can be assumed and that transmit filters in the UL are appropriately designed. In this way the estimation of the interfering channels is avoided, while we can take advantage of both DL transmit coordination to adjust transmit power and beamformers and the use of improper Gaussian signaling to exploit the real and imaginary dimensions of the MIMO channel. Simulations show that the proposed decentralized technique allows reducing the mean square error (MSE) and increasing user throughput in highly interfered scenarios.**

Keywords—improper Gaussian signaling; MIMO interference channel; decentralized widely linear precoding design; MMSE.

I. Introduction

The multiple-input multiple-output interference channel (MIMO-IC) is a generic model for multiuser cellular communication systems, in which a number of transmitters, each equipped with multiple antennas, wish to simultaneously send independent messages to its intended receiver while generating interference to all remaining receivers. Unfortunately, the optimal transmit/receive strategy with linear filters that maximizes the sum-rate of the system is not known because of the non-convexity of the problem. In this regard, a line of current research focuses on the coordination among transmitters based on the concept of the *interference cost* introduced in [1][2], which achieves a local optimum sum-rate solution and manages the interference in a decentralized manner. On the other hand, the maximization of the sum-rate can be obtained by iteratively minimizing the weighted mean square error (MSE), see [3], where the weighting matrices are chosen accordingly to the inherent relation between the achievable rate and the MSE [4]. Further, in [5], we describe decentralized procedures that avoid estimation and reporting of the interfering channels by exploiting the received signal in the UL in case propagation channel reciprocity can be assumed, while they get a local optimum of the maximum sum-rate or minimum MSE (MMSE) problem. Its practical implementation in 3GPP LTE-Advanced was analyzed in [6], showing important throughput gains.

In all these works and related ones it is assumed that the transmitted signals are *proper* (or circularly symmetric complex) Gaussian distributed. Even though proper Gaussian signaling has been shown to be optimal in terms of capacity for the Gaussian point-to-point (P2P), multiple-access channel (MAC) and broadcast channel (BC), for Gaussian interference channels (IC) with the interference treated as noise, *improper* Gaussian signaling is able to improve the known achievable rates [7]. The key difference between proper and improper Gaussian signaling is the characterization of the second order statistics: while proper Gaussian random vectors are fully specified by the covariance matrices under the zero-mean assumption, improper Gaussian random vectors are characterized by the covariance matrices and *pseudo-covariance* matrices [8][9].
Existing approaches in the literature [10][11] tackle the complex-valued MIMO-IC with improper Gaussian signaling through the reformulation of the problem into an equivalent double-sized real-valued MIMO-IC by separating the real and imaginary parts of the channel matrix. This way, most of the approaches already developed for MIMO-IC with proper Gaussian signaling can be applied, but much of the insight is lost. Thanks to the use of the complex-valued channel model, in [12] it is shown that for any given set of covariance matrices, the achievable rates for MIMO-IC can be improved with improper Gaussian signaling by optimizing the transmit pseudo-covariance matrices. In this regard, transmit covariance and pseudo-covariance optimization techniques are proposed for the two-user SISO-IC in [12] and for the K-user MISO-IC in [13]. Additionally, in [14], we show that an MSE-based beamforming design could be useful to increase the system performance and fairness in terms of bitrate in the MIMO Z-IC (a particular two-user MIMO-IC scenario).

An effective way to generate and estimate improper Gaussian signals is by means of using widely linear precoding (WLP) and widely linear estimation (WLE) [8], respectively. WLE receivers are already under investigation in 3GPP LTE-A [15], however, little has been done regarding the design of WLP at transmitters. Meanwhile the improper-based schemes derived in [12][13] are not valid for the multi-antenna receiver case, the proper-based schemes developed in [5] cannot be applied to a scenario with heterogeneous types of receivers. So, new WLP

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designs are required for the MIMO-IC.

In this article, the most general K-user MIMO-IC is considered and we focus on managing interference of a downlink (DL) cellular network in a decentralized manner when improper Gaussian signaling is applied. DL transmit coordination for decentralized WLP design is attained by using the interference cost concept and MMSE criterion. The proposed scheme exploits the UL transmission as in [5], however, the major differences come due to the use of improper Gaussian signaling and are the following ones: *i*) new transmit beamformers are derived, *ii*) two interference cost matrices are needed for DL transmit coordination, and *iii*) the exploitation of the UL transmission requires a different scheme.

The contributions of this work are:

- Minimum MSE problem is formulated using the complex-valued channel model, which allows covering a backwards compatibility-oriented scenario where different types of users coexist: ones that are able to apply WLE while others which are equipped with conventional linear estimation (LE) receivers, which may be found in a real-based deployment [15].
- A decentralized and scalable transmit/receive beamforming design using improper Gaussian signaling is presented. The proposed procedure exploits the received signal in the UL, provided that propagation channel reciprocity can be assumed. The key aspect in the UL transmission is that users applying WLE in the DL need to transmit improper Gaussian signaling (i.e. use WLP) in the UL.
- It is shown that both the MSE and the transmission rate are enhanced with transmit coordination and improper Gaussian signaling as compared to conventional interference management procedures with proper Gaussian signaling, and as compared to improper Gaussian signaling schemes without DL transmit coordination. Simulations allow identifying the scenarios where the gains are largest.

II. Improper Gaussian Signaling for MIMO-IC

Consider an interference channel (IC) of K transmitters equipped with M_k antennas, each serving a single receiver with N_k receive antenna elements (k=1,…,K). The MIMO channel between the j-th transmitter and the k-th receiver is described by a matrix $\mathbf{H}_k^j \in \mathbb{C}^{N_k \times M_j}$ containing the complex-valued channel gains of the different antenna-pairs. Hence, assuming narrow-band transmissions, the equivalent baseband signal observed at the k-th receiver can be expressed as:

$$\mathbf{y}_k = \mathbf{H}_k^k \mathbf{x}_k + \mathbf{s}_k = \mathbf{H}_k^k \mathbf{x}_k + \sum_{\substack{j=1 \\ j\neq k}}^{K} \mathbf{H}_k^j \mathbf{x}_j + \mathbf{n}_k \quad k=1,...,K \ , \quad (1)$$

where $\mathbf{x}_k \in \mathbb{C}^{M_k \times 1}$ is the complex-valued transmitted vector and $\mathbf{s}_k$ denotes the received interference-plus-noise at the k-th receiver, which is composed by interference and circularly symmetric white Gaussian (i.e. proper) noise with distribution $\mathbf{n}_k \sim CN(\mathbf{0}, \sigma_k^2 \mathbf{I})$. Different from the conventional transmission setup where proper Gaussian signaling is assumed (i.e. $\mathbf{x}_k \sim CN(\mathbf{0}, \mathbf{C}_{\mathbf{x}_k})$), in this paper the most general *improper* Gaussian signaling is used, in which the second order statistics of $\mathbf{x}_k$ are given by the covariance matrix $\mathbf{C}_{\mathbf{x}_k} = E\{\mathbf{x}_k \mathbf{x}_k^H\}$ and also the pseudo-covariance matrix $\tilde{\mathbf{C}}_{\mathbf{x}_k} = E\{\mathbf{x}_k \mathbf{x}_k^T\}$ [8].

An efficient way to map a proper Gaussian information-bearing signal $\mathbf{b}_k$ into an improper signal $\mathbf{x}_k$, is by means of using *widely linear precoding* (WLP) [12] as follows:

$$\mathbf{x}_k = \mathbf{T}_k^1 \mathbf{b}_k + \mathbf{T}_k^2 \mathbf{b}_k^* \quad k=1,...,K \ , \quad (2)$$

where matrices $\mathbf{T}_k^1$ and $\mathbf{T}_k^2 \in \mathbb{C}^{M_k \times m_k}$ are the transmit linear filters (or beamformers) for the information-bearing signal $\mathbf{b}_k \in \mathbb{C}^{m_k \times 1}$ and its complex conjugate $\mathbf{b}_k^*$, respectively, where $m_k \le \min(N_k, M_k)$ denotes the number of transmission modes toward the k-th receiver. It is assumed that $\mathbf{b}_k$ is a proper Gaussian random vector (i.e. $\mathbf{b}_k \sim CN(\mathbf{0}, \mathbf{I})$) and that it is independent of $\mathbf{b}_k^*$ and all other $\mathbf{b}_j$ for all $j\neq k$. Then, the covariance and pseudo-covariance matrices of the improper transmitted signal $\mathbf{x}_k$ towards the k-th receiver are given by:

$$\mathbf{C}_{\mathbf{x}_k} = \mathbf{T}_k^1 \mathbf{T}_k^{1H} + \mathbf{T}_k^2 \mathbf{T}_k^{2H}, \qquad \tilde{\mathbf{C}}_{\mathbf{x}_k} = \mathbf{T}_k^1 \mathbf{T}_k^{2T} + \mathbf{T}_k^2 \mathbf{T}_k^{1T} \ . \quad (3)$$

The conventional setup with proper Gaussian signaling is a special case of improper signaling transmission with $\tilde{\mathbf{C}}_{\mathbf{x}_k} = \mathbf{0}$, which can be achieved by setting $\mathbf{T}_k^2 = \mathbf{0}$ and using the conventional *linear precoding* (LP) scheme in (2): $\mathbf{x}_k = \mathbf{T}_k^1 \mathbf{b}_k$.

In order to access the information contained in the pseudo-covariance matrices, *widely linear estimation* (WLE) needs to be applied at the receiver [8]. Hence, symbols towards the k-th receiver are estimated according to:

$$\hat{\mathbf{b}}_k = \mathbf{R}_k^{1H} \mathbf{y}_k + \mathbf{R}_k^{2H} \mathbf{y}_k^* \quad k=1,...,K \ , \quad (4)$$

where matrices $\mathbf{R}_k^1$ and $\mathbf{R}_k^2 \in \mathbb{C}^{N_k \times m_k}$ are the linear receive filters. Notice that the well-known *linear estimation* (LE) is a particular case of WLE in which $\hat{\mathbf{b}}_k = \mathbf{R}_k^{1H} \mathbf{y}_k$.

The mean square error (MSE) for the symbols transmitted towards the k-th receiver can be expressed through the so-called *MSE-matrix* $\mathbf{E}_k = E\{\mathbf{e}_k \mathbf{e}_k^H\}$, where $\mathbf{e}_k = \hat{\mathbf{b}}_k - \mathbf{b}_k$. As the different streams of transmitted symbols $\mathbf{b}_k$ corresponds to independent and proper random vectors (and hence uncorrelated), the MSE-matrix for the k-th receiver can be developed in terms of the transmit and receive filters in (2) and (4) as:

$$\begin{aligned} \mathbf{E}_k = {} & \mathbf{I} - \mathbf{R}_k^{1H} \mathbf{H}_k^k \mathbf{T}_k^1 - \mathbf{T}_k^{1H} \mathbf{H}_k^{kH} \mathbf{R}_k^1 - \mathbf{R}_k^{2H} \mathbf{H}_k^{k*} \mathbf{T}_k^{2*} - \mathbf{T}_k^{2T} \mathbf{H}_k^{kT} \mathbf{R}_k^2 \\ & + \mathbf{R}_k^{1H} \mathbf{C}_{\mathbf{y}_k} \mathbf{R}_k^1 + \mathbf{R}_k^{2H} \mathbf{C}_{\mathbf{y}_k}^* \mathbf{R}_k^2 + \mathbf{R}_k^{1H} \tilde{\mathbf{C}}_{\mathbf{y}_k} \mathbf{R}_k^2 + \mathbf{R}_k^{2H} \tilde{\mathbf{C}}_{\mathbf{y}_k}^* \mathbf{R}_k^1 \end{aligned} , \quad (5)$$

where $\mathbf{C}_{\mathbf{y}_k}$ and $\tilde{\mathbf{C}}_{\mathbf{y}_k}$ denote the covariance and pseudo-covariance matrices of the received signal $\mathbf{y}_k$ in (1):

$$\begin{aligned} \mathbf{C}_{\mathbf{y}_k} &= \mathbf{H}_k^k \mathbf{C}_{\mathbf{x}_k} \mathbf{H}_k^{kH} + \mathbf{C}_{\mathbf{s}_k}, \ \ \mathbf{C}_{\mathbf{s}_k} = \sum_{\substack{j=1 \\ j\neq k}}^{K} \mathbf{H}_k^j \mathbf{C}_{\mathbf{x}_j} \mathbf{H}_k^{jH} + \sigma_k^2 \mathbf{I} \\ \tilde{\mathbf{C}}_{\mathbf{y}_k} &= \mathbf{H}_k^k \tilde{\mathbf{C}}_{\mathbf{x}_k} \mathbf{H}_k^{kT} + \tilde{\mathbf{C}}_{\mathbf{s}_k}, \ \ \tilde{\mathbf{C}}_{\mathbf{s}_k} = \sum_{\substack{j=1 \\ j\neq k}}^{K} \mathbf{H}_k^j \tilde{\mathbf{C}}_{\mathbf{x}_j} \mathbf{H}_k^{jT} \end{aligned} , \quad (6)$$

being $\mathbf{C}_{\mathbf{s}_k}$ and $\tilde{\mathbf{C}}_{\mathbf{s}_k}$ the covariance and pseudo-covariance matrices of the interference-plus-noise term $\mathbf{s}_k$ in (1).

Under the assumption that interference is treated as Gaussian noise, the achievable rate for the k-th receiver is [12]:

$$R_k = \log\left(\frac{\left|\mathbf{C}_{\mathbf{y}_k}\right|}{\left|\mathbf{C}_{\mathbf{s}_k}\right|}\right) + \frac{1}{2}\log\left(\frac{\left|\mathbf{I} - \mathbf{C}_{\mathbf{y}_k}^{-1} \tilde{\mathbf{C}}_{\mathbf{y}_k} \mathbf{C}_{\mathbf{y}_k}^{-T} \tilde{\mathbf{C}}_{\mathbf{y}_k}^H\right|}{\left|\mathbf{I} - \mathbf{C}_{\mathbf{s}_k}^{-1} \tilde{\mathbf{C}}_{\mathbf{s}_k} \mathbf{C}_{\mathbf{s}_k}^{-T} \tilde{\mathbf{C}}_{\mathbf{s}_k}^H\right|}\right) . \quad (7)$$

III. Decentralized Interference Management with Improper Gaussian Signaling

Let us focus on a DL cellular network, where transmitters are base stations (BSs) while receivers are user equipments (UEs) (although concepts can be easily applied for the UL). When adopting an MMSE criterion, the problem of interest is to find transmit and receive filters (or beamformers) such that the sum of MSEs is minimized while the power budget of each BS is respected. As $\mathbf{b}_k$ in (2) is a proper Gaussian random vector, the power transmitted by the k-th BS is given by $P_k = \mathrm{Tr}\,(\mathbf{T}_k^1\mathbf{T}_k^{1H} + \mathbf{T}_k^2\mathbf{T}_k^{2H})$. Hence, transmit and receive filters are obtained as solution of the following optimization problem:

$$\begin{aligned}(\mathrm{P}_0):\quad & \underset{\{\mathbf{T}_k^1\},\{\mathbf{T}_k^2\},\{\mathbf{R}_k^1\},\{\mathbf{R}_k^2\}}{\text{minimize}} \quad \sum\nolimits_{k=1}^{K} \mathrm{Tr}\,(\mathbf{E}_k) \\ & \text{subject to} \quad \mathrm{Tr}\,(\mathbf{T}_k^1\mathbf{T}_k^{1H} + \mathbf{T}_k^2\mathbf{T}_k^{2H}) \le P_k^{DL} \quad k=1,\dots,K\end{aligned}, \qquad (8)$$

where $\mathbf{E}_k$ denotes the MSE-matrix defined in (5), and P_k^{DL} is the maximum power available at the k-th BS. Although problem (P_0) in (8) is not jointly convex on all sets of transmit and receive filters, it is convex for each set of variables separately $\{\mathbf{T}_k^1\},\{\mathbf{T}_k^2\},\{\mathbf{R}_k^1\},\{\mathbf{R}_k^2\}$. So, each of them can be obtained by deriving the Lagrangian function of problem (P_0) in (8) (see details in Appendix VI) assuming that the rest sets of variables are fixed. Furthermore, by working with the derived expressions, the optimal transmit filters $\{\mathbf{T}_k^1,\mathbf{T}_k^2\}$ can be obtained for a set of fixed receive filters $\{\mathbf{R}_k^1,\mathbf{R}_k^2\}$, and the other way round. The result is given in Proposition 1.

Let us define the following parameters which will allow us to compact the transmit filter (beamformer) design:

$$\begin{aligned}\mathrm{A}_k &= \mathbf{H}_k^{kH}(\mathbf{R}_k^1\mathbf{R}_k^{1H} + \mathbf{R}_k^{2*}\mathbf{R}_k^{2T})\mathbf{H}_k^k + \boldsymbol{\Upsilon}_k \\ \mathrm{B}_k &= \mathbf{H}_k^{kH}(\mathbf{R}_k^1\mathbf{R}_k^{2H} + \mathbf{R}_k^{2*}\mathbf{R}_k^{1T})\mathbf{H}_k^{k*} + \boldsymbol{\Gamma}_k\end{aligned}, \qquad (9)$$

where matrices $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ will be referred as *interference cost matrix* and *pseudo-interference cost matrix*, respectively:

$$\begin{aligned}\boldsymbol{\Upsilon}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} \mathbf{H}_j^{kH}(\mathbf{R}_j^1\mathbf{R}_j^{1H} + \mathbf{R}_j^{2*}\mathbf{R}_j^{2T})\mathbf{H}_j^k \\ \boldsymbol{\Gamma}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} \mathbf{H}_j^{kH}(\mathbf{R}_j^1\mathbf{R}_j^{2H} + \mathbf{R}_j^{2*}\mathbf{R}_j^{1T})\mathbf{H}_j^{k*}\end{aligned}. \qquad (10)$$

Proposition 1: The solution to problem (P_0) in (8) is given by:

$$\begin{aligned}\mathbf{T}_k^1 &= \mathbf{F}_k^{-1}(\mathbf{H}_k^{kH}\mathbf{R}_k^1 - \mathbf{B}_k(\mathbf{A}_k + \lambda_k\mathbf{I})^{-1*}\mathbf{H}_k^{kT}\mathbf{R}_k^2) \\ \mathbf{T}_k^2 &= \mathbf{F}_k^{-1}(\mathbf{H}_k^{kH}\mathbf{R}_k^{2*} - \mathbf{B}_k(\mathbf{A}_k + \lambda_k\mathbf{I})^{-1*}\mathbf{H}_k^{kT}\mathbf{R}_k^{1*}) \\ \mathbf{R}_k^1 &= \begin{cases}\mathbf{D}_{\mathbf{y}_k}^{-1}(\mathbf{H}_k^k\mathbf{T}_k^1 - \tilde{\mathbf{C}}_{\mathbf{y}_k}\mathbf{C}_{\mathbf{y}_k}^{-1*}\mathbf{H}_k^{k*}\mathbf{T}_k^{2*}) & \text{if WLE} \\ \mathbf{C}_{\mathbf{y}_k}^{-1}\mathbf{H}_k^k\mathbf{T}_k^1 & \text{if LE}\end{cases} \\ \mathbf{R}_k^2 &= \begin{cases}\mathbf{D}_{\mathbf{y}_k}^{-1*}(\mathbf{H}_k^{k*}\mathbf{T}_k^{2*} - \tilde{\mathbf{C}}_{\mathbf{y}_k}^{*}\mathbf{C}_{\mathbf{y}_k}^{-1}\mathbf{H}_k^k\mathbf{T}_k^1) & \text{if WLE} \\ \mathbf{0} & \text{if LE}\end{cases}\end{aligned}, \qquad (11)$$

where λ_k is the non-negative dual variable associated to the k-th power constraint in (8), and

$$\begin{aligned}\mathbf{D}_{\mathbf{y}_k} &= \mathbf{C}_{\mathbf{y}_k} - \tilde{\mathbf{C}}_{\mathbf{y}_k}\mathbf{C}_{\mathbf{y}_k}^{-1*}\tilde{\mathbf{C}}_{\mathbf{y}_k}^{*} \\ \mathbf{F}_k &= \mathbf{A}_k + \lambda_k\mathbf{I} - \mathbf{B}_k(\mathbf{A}_k + \lambda_k\mathbf{I})^{-1*}\mathbf{B}_k^{*}\end{aligned}. \qquad (12)$$

Proof: See Appendix VI.

As pointed out before, the formulation with the complex-valued channel model to solve problem in (8) allows taking into account a scenario with different types of receivers (WLE and LE), which might be found in a backwards compatible deployment. Receive filters in (11) are already under study in 3GPP LTE-Advanced standard [15], where they are referred to as LMMSE and WLMMSE. The novelty here is the derivation of the widely linear transmit filters $\{\mathbf{T}_k^1,\mathbf{T}_k^2\}$ and the joint optimization. It can be proven by means of the Schur complement property [16] that $\mathbf{F}_k$ in (12) is an invertible matrix, since $\mathbf{A}_k + \lambda_k\mathbf{I}$ is a full rank matrix.

Let us recall that in a point-to-point communication if the user is equipped with LE, the optimal transmission scheme in terms of MSE is given by LP (i.e. $\mathbf{T}_k^2 = \mathbf{0}$), which can be easily established from the MSE-matrix in (5). However, this may not be further the case in a MIMO-IC, as the use of WLP can be useful to improve a global utility function as in (8).

As transmit and receive filters in (11) cannot be solved analytically, we can follow a block coordinate descent approach to find a local optimum to problem (P_0) in (8) by alternate optimization of transmit and receive filters in (11). This procedure is fully centralized, nevertheless, a decentralized approach could be derived if every BS had knowledge of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ in (10).

Assuming that each k-th BS has knowledge of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ in (10) and $\mathbf{C}_{\mathbf{s}_k}$ and $\tilde{\mathbf{C}}_{\mathbf{s}_k}$ in (6), problem (P_0) in (8) is equivalent to define the following problem to be solved at each k-th BS:

$$\begin{aligned}(\mathrm{P}_{0,k}):\quad & \underset{\mathbf{T}_k^1,\mathbf{T}_k^2,\mathbf{R}_k^1,\mathbf{R}_k^2}{\text{minimize}} \quad \mathrm{Tr}\,(\mathbf{E}_k) + g_{-k} \\ & \text{subject to} \quad \mathrm{Tr}\,(\mathbf{T}_k^1\mathbf{T}_k^{1H} + \mathbf{T}_k^2\mathbf{T}_k^{2H}) \le P_k^{DL}\end{aligned}, \qquad (13)$$

where function g_{-k} stands for the impact of the k-th BS over the MSE of the UEs associated to the neighboring BSs, which depends on $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ defined in (10) as:

$$g_{-k} = \mathrm{Tr}\,(\boldsymbol{\Upsilon}_k(\mathbf{T}_k^1\mathbf{T}_k^{1H} + \mathbf{T}_k^2\mathbf{T}_k^{2H})) + \mathrm{Tr}\,(\boldsymbol{\Gamma}_k(\mathbf{T}_k^{2*}\mathbf{T}_k^{1H} + \mathbf{T}_k^1\mathbf{T}_k^{2T})) \qquad (14)$$

Problem ($\mathrm{P}_{0,k}$) in (13) is convex on transmit $(\mathbf{T}_k^1,\mathbf{T}_k^2)$ and receive $(\mathbf{R}_k^1,\mathbf{R}_k^2)$ filters separately, and leads to the solution in (11). So, for given $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ in (10) and $\mathbf{C}_{\mathbf{s}_k}$ and $\tilde{\mathbf{C}}_{\mathbf{s}_k}$ in (6), each k-th BS can solve problem ($\mathrm{P}_{0,k}$) in (13) with alternate optimization between transmit and receive filters in (11). Compared to the decentralized procedure presented in [5] for the proper Gaussian signaling case, when improper Gaussian signaling is used two interfering cost matrices are required ($\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$). A possible solution to obtain them is by exchanging control-plane messages among BSs, since $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ can be decomposed as the sum of information from neighboring cells. However, with such approach, BSs need knowledge of all the interfering channel matrices and local receive filters (see (10)).

In order to avoid the complex task of estimating and reporting the interfering channels by the UEs, it is proposed to obtain an estimate of the interference cost $\boldsymbol{\Upsilon}_k$ and the pseudo-interference cost $\boldsymbol{\Gamma}_k$ in (10) at each k-th BS from the covariance matrix and the pseudo-covariance matrix of the UL received signal, respectively, by exploiting UL-DL propagation channel reciprocity and appropriately designing UL transmit filters by means of WLP. By doing so, errors associated to the estimation of the interfering channels would be avoided and its impact on beamforming design would be reduced. Assuming UL-DL propagation channel reciprocity (as in a TDD mode with slowly varying channel for duplexing UL and DL), the covariance and pseudo-covariance matrices of the received UL interference-plus-noise signal at the k-th BS are:

$$\begin{aligned}\overleftarrow{\boldsymbol{\Psi}}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} \mathbf{H}_j^{kT}(\overleftarrow{\mathbf{T}}_j^1\overleftarrow{\mathbf{T}}_j^{1H} + \overleftarrow{\mathbf{T}}_j^2\overleftarrow{\mathbf{T}}_j^{2H})\mathbf{H}_j^{k*} + \overleftarrow{\sigma}_k^2\mathbf{I} \\ \overleftarrow{\tilde{\boldsymbol{\Psi}}}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} \mathbf{H}_j^{kT}(\overleftarrow{\mathbf{T}}_j^1\overleftarrow{\mathbf{T}}_j^{2T} + \overleftarrow{\mathbf{T}}_j^2\overleftarrow{\mathbf{T}}_j^{1T})\mathbf{H}_j^k\end{aligned}, \qquad (15)$$

where $\overleftarrow{\mathbf{T}}_j^1$ and $\overleftarrow{\mathbf{T}}_j^2$ denote the UL transmit filters at the j-th UE and $\overleftarrow{\sigma}_k^2$ is the UL noise power. Notice that $\boldsymbol{\Upsilon}_k$ in (10) and $\overleftarrow{\boldsymbol{\Psi}}_k^*$ in (15) differ just in the noise term while $\boldsymbol{\Gamma}_k$ in (10) and $\overleftarrow{\boldsymbol{\Psi}}_k^*$ in (15) are equivalent in case that $\overleftarrow{\mathbf{T}}_j^1 = \mathbf{R}_j^{1*}$ and $\overleftarrow{\mathbf{T}}_j^2 = \mathbf{R}_j^2$ for all users. Nevertheless, uplink transmit filters cannot be applied as such unless we take into account the constraints imposed by the maximum transmit power for UL transmissions at UEs, i.e.

$$\mathrm{Tr}\,(\overleftarrow{\mathbf{T}}_k^1\overleftarrow{\mathbf{T}}_k^{1H} + \overleftarrow{\mathbf{T}}_k^2\overleftarrow{\mathbf{T}}_k^{2H}) \le P_k^{UL} \qquad k = 1,\ldots,K \ , \tag{16}$$

where P_k^{UL} is the available power at the k-th UE. In this regard, we propose to scale by a common cell-wide factor F the receive filters obtained as solution of problem in (13) in order to satisfy the UL power constraint:

$$\overleftarrow{\mathbf{T}}_k^1 = \sqrt{F}\mathbf{R}_k^{1*}, \quad \overleftarrow{\mathbf{T}}_k^2 = \sqrt{F}\mathbf{R}_k^2 \ , \tag{17}$$

where parameter $F \le 1$ is designed beforehand in order to ensure that the ε-% of the UEs fulfill the transmit power constraint, i.e.

$$\Pr\left\{\mathrm{Tr}\,(F\mathbf{R}_k^{1*}\mathbf{R}_k^{1T} + F\mathbf{R}_k^2\mathbf{R}_k^{2H}) \le P_k^{UL}\right\} = \varepsilon \ . \tag{18}$$

It is assumed that those users not satisfying the constraint in (16) with the a priori selected F, will transmit at maximum power by scaling the receive filters in (17) with a factor $f_k < F$:

$$\begin{gathered}\overleftarrow{\mathbf{T}}_k^1 = \sqrt{f_k}\mathbf{R}_k^{1*}, \ \overleftarrow{\mathbf{T}}_k^2 = \sqrt{f_k}\mathbf{R}_k^2, \\ f_k = P_k^{UL} \big/ \mathrm{Tr}\,(\mathbf{R}_k^{1*}\mathbf{R}_k^{1T} + \mathbf{R}_k^2\mathbf{R}_k^{2H})\end{gathered} \ . \tag{19}$$

By using this approach for designing the UL transmit filters, we can employ the received signal in the UL to get an approximation of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ in (10) as follows:

$$\begin{gathered}\hat{\boldsymbol{\Upsilon}}_k = F^{-1}\overleftarrow{\boldsymbol{\Psi}}_k^* = \boldsymbol{\Upsilon}_k + \tilde{\boldsymbol{\Upsilon}}_k + F^{-1}\overleftarrow{\sigma}_k^2\mathbf{I} \\ \hat{\boldsymbol{\Gamma}}_k = F^{-1}\overleftarrow{\tilde{\boldsymbol{\Psi}}}_k^* = \boldsymbol{\Gamma}_k + \tilde{\boldsymbol{\Gamma}}_k\end{gathered} \ , \tag{20}$$

where $\tilde{\boldsymbol{\Upsilon}}_k$ and $\tilde{\boldsymbol{\Gamma}}_k$ describe the error introduced by those users in neighbor cells transmitting at maximum power (see (18)):

$$\begin{aligned}\tilde{\boldsymbol{\Upsilon}}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} F^{-1}(f_j - F)\mathbf{H}_j^{kH}(\mathbf{R}_j^1\mathbf{R}_j^{1H} + \mathbf{R}_j^{2*}\mathbf{R}_j^{2T})\mathbf{H}_j^k \\ \tilde{\boldsymbol{\Upsilon}}_k &= \sum\nolimits_{\substack{j=1\\ j\neq k}}^{K} F^{-1}(f_j - F)\mathbf{H}_j^{kH}(\mathbf{R}_j^1\mathbf{R}_j^{2H} + \mathbf{R}_j^{2*}\mathbf{R}_j^{1T})\mathbf{H}_j^{k*}\end{aligned} \ . \tag{21}$$

Estimators in (20) are biased due to the transmissions of those users that are transmitting at maximum power in the UL, and $\hat{\boldsymbol{\Upsilon}}_k$ has an additional bias that depends on the UL noise power increased by F^{-1}, which does not affect the estimation of $\hat{\boldsymbol{\Gamma}}_k$ due to the proper statistics of the noise. The adequate selection of the scaling factor F is very important, as it is shown in [5]. However, all what is needed to obtain $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ from the UL transmission are the UL received signal but not the information symbols, so an UL pilot-based transmission could be used.

Algorithm I summarizes the decentralized procedure to solve problem (P_0) in (8). The procedure embeds the optimization of problem ($P_{0,k}$) in (13) at each k-th BS, that is done by using a block coordinate descent approach with alternate optimization between transmit and receive filters in (11), and local convergence is ensured [3]. Convergence of Algorithm I cannot be guaranteed due to the errors in the estimation of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$, although it has been observed in simulations (see Section IV). If matrices $\hat{\boldsymbol{\Upsilon}}_k$ and $\hat{\boldsymbol{\Gamma}}_k$ were estimated without errors, monotonic convergence is guaranteed with the rationale in [5].

Algorithm I: Decentralized algorithm for WLP design

1. *Initialize*: $\{\mathbf{T}_k^1, \mathbf{T}_k^2\}$
2. *Set*: $\{\hat{\boldsymbol{\Upsilon}}_k = \mathbf{0}, \hat{\boldsymbol{\Gamma}}_k = \mathbf{0}\}$, $t = 0$, and evaluate $\{\mathbf{C}_{\mathbf{s}_k}, \tilde{\mathbf{C}}_{\mathbf{s}_k}\}$ (6)
3. *Do*: - (**#DL Transmission**) For each transmitter $k = 1,\ldots,K$:
   $[(\mathbf{T}_k^1, \mathbf{T}_k^2)^{t+1}, (\mathbf{R}_k^1, \mathbf{R}_k^2)^{t+1}]$ = Solve ($P_{0,k}$) using $(\hat{\boldsymbol{\Upsilon}}_k, \hat{\boldsymbol{\Gamma}}_k)^t, (\mathbf{C}_{\mathbf{s}_k}, \tilde{\mathbf{C}}_{\mathbf{s}_k})^t$
4. - For each user: evaluate $(\mathbf{C}_{\mathbf{s}_k}, \tilde{\mathbf{C}}_{\mathbf{s}_k})^{t+1}$ (6) and update $(\mathbf{R}_k^1, \mathbf{R}_k^2)^{t+1}$ (11)
5. - Set UL transmit filters: $(\overleftarrow{\mathbf{T}}_k^1, \overleftarrow{\mathbf{T}}_k^2)^{t+1} = (\sqrt{F}\mathbf{R}_k^{1*}, \sqrt{F}\mathbf{R}_k^2)^{t+1}$
6. - (**#UL Transmission**) All UE transmit simultaneously using $\{\overleftarrow{\mathbf{T}}_k^1, \overleftarrow{\mathbf{T}}_k^2\}^{t+1}$
7. - Each transmitter estimates $(\hat{\boldsymbol{\Upsilon}}_k, \hat{\boldsymbol{\Gamma}}_k)^{t+1}$ (20) and updates $(\mathbf{T}_k^1, \mathbf{T}_k^2)^{t+1}$(11)
8. - $t \leftarrow t+1$
9. ... *until convergence*

IV. Evaluation and Results

In this section we evaluate the performance of the decentralized transmit and receive beamformers optimization proposed in Section III through Montecarlo simulations. This procedure is compared to the well-known centralized alternate MMSE optimization [3] and other decentralized solutions presented in [5] with proper Gaussian signaling. A MIMO-IC composed of K transmitters (each with a single associated user) is considered. It is assumed that all transmitters have the same available power P^{DL}. Signal-to-noise ratio is defined as SNR=P^{DL}/σ^2, and signal-to-interference ratio is given by SIR=$1/(\eta(K\text{-}1))$, where factor $0<\eta\le 1$ denotes the average ratio between interfering and direct channel attenuations. Channels are modeled through a Rayleigh distribution and 100 different channel realizations are evaluated. Antenna configurations are depicted in figures as $M_k \times N_k$.

The following techniques are evaluated, both for the case of *proper* (i.e. LP and LE) and *improper* (i.e. WLP and WLE):

- *cent-IM*: centralized interference management (IM) technique that consists on alternate optimization between transmit and receive filters in (11) in a central processor node. Infinite backhaul bandwidth and perfect CSI for all links is needed to get and exchange all interfering channel matrices.
- *decent-IM*: proposed decentralized IM procedure in Section III that solves ($P_{0,k}$) in (13) with Algorithm I by exploiting propagation channel reciprocity to get an estimate of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$. Perfect CSI is needed only for the direct link.
- *decent-IM ideal*: decentralized IM technique in Algorithm I with the exact values of $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$ (no estimation errors).
- *no-IM*: No interference management is performed. Each BS optimizes its filters to combat the received noise-plus-interference at the attached UE, solving ($P_{0,k}$) in (13) with $\boldsymbol{\Upsilon}_k = \boldsymbol{\Gamma}_k = \mathbf{0}$. Perfect CSI is needed for the direct link.

The number of iterations in the decentralized procedures is limited to 10. The best result among 5 random transmit initializations is considered for all MMSE-based optimizations. For the improper cases, a WLP initialization is required (i.e. random $\{\mathbf{T}_k^1, \mathbf{T}_k^2\}$ such that the power budget at each k-th BS is respected), otherwise the proposed procedure leads to a proper scheme. The performance indicators are the *per-user MSE* and the *mean user throughput* (UT) measured in bits/s/Hz.

Fig. 1 shows the convergence of the proposed decentralized IM techniques in terms of per-user MSE, for K=2 and K=5, 1×1 antenna configuration, when using η=1, and SNR=20dB. The solution achieved with the 'cent-IM' procedures is plotted with dashed lines in Fig. 1. It can be observed that 'proper/improper decent-IM ideal' have a monotonic convergence, while the proposed 'proper/improper decent-IM' procedures based on propagation channel reciprocity also converge, although the convergence is observed to be non-monotonic in some specific realizations due to the errors in the estimation of the matrices $\boldsymbol{\Upsilon}_k$ and $\boldsymbol{\Gamma}_k$. The convergence is in general shown to be faster for the proper-based procedures.

Fig. 2 displays the per-user MSE vs. the number of transmit-receive pairs (K) when using η=1 and η=0.5, for antenna configuration 1×1 or 2×2, and SNR=20dB. As it is expected, the per-user MSE is reduced as K increases due to the augmented interference. It can be observed that, for both

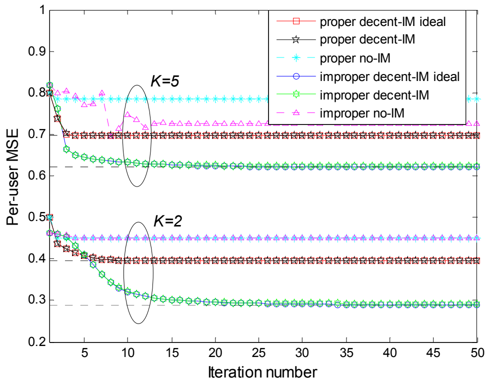
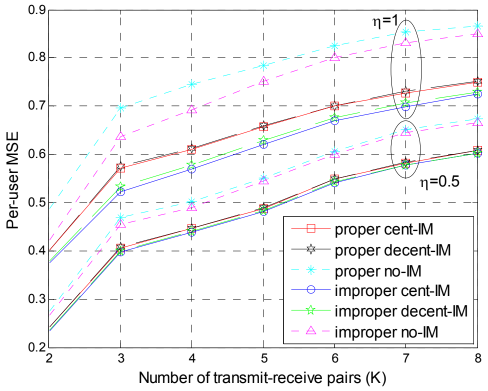
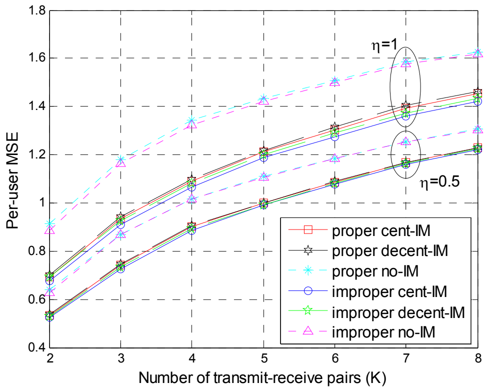


Fig. 1: Convergence of the proposed decentralized IM techniques in a specific channel realization for K=2 and K=5, η=1, 1×1, SNR=20dB.



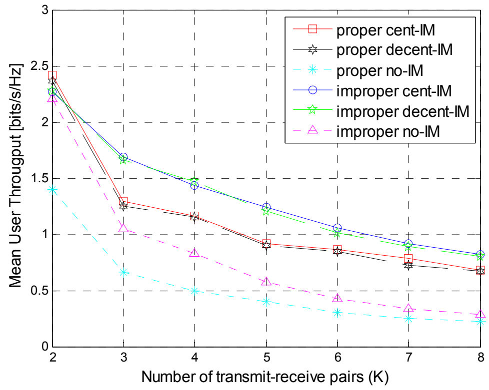
a) Antenna configuration: 1×1.



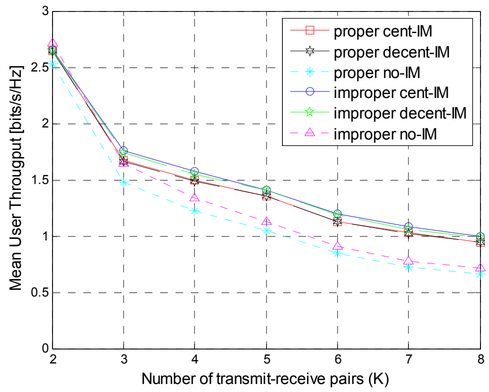
b) Antenna configuration: 2×2.

Fig. 2: Per-user mean square error (MSE) vs. number of transmit-receive pairs (K) for η=1 and η=0.5, antenna configuration 1×1 or 2×2, SNR=20dB.

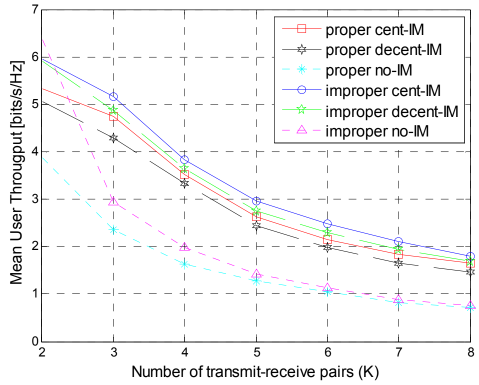
antenna configurations, the use of ‘improper no-IM’ outperforms ‘proper no-IM’ in terms of MSE, and the use of IM with improper (either ‘improper cent-IM’ or ‘improper decent-IM’) outperforms the IM with proper (either ‘proper cent-IM’ or ‘proper decent-IM’). The MSE reduction provided by IM techniques is evident and significant for all antenna configurations and number of potential interferers (K). The MSE reduction provided by the use of improper Gaussian signaling is larger for η=1 than for η=0.5 as the interference to be managed is stronger, and it is larger for the 1×1 antenna configuration rather than for the 2×2 case because the use of improper Gaussian signaling provides diversity (or an extra dimension) which already exists in the 2×2 antenna configuration. For that reason, the gains of IM and improper in



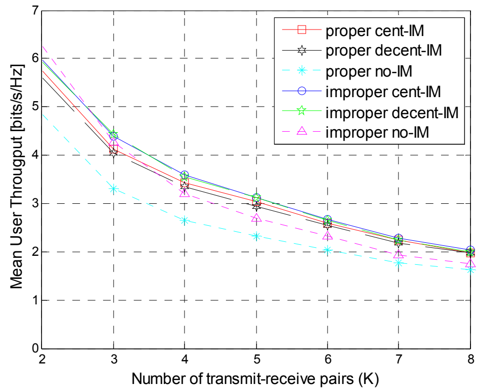
a) Antenna configuration: 1×1, η=1.



b) Antenna configuration: 1×1, η=0.5.



c) Antenna configuration: 2×2, η=1.



d) Antenna configuration: 2×2, η=0.5.

Fig. 3: Mean user throughput (UT) (bits/s/Hz) vs. number of transmit-receive pairs (K) for η=1 or 0.5, antenna configuration 1×1 or 2×2, SNR=20dB.

the 2×2 case are appreciable when the number of potential interferers is large, while in the 1×1 case they are substantial even for a scenario with a single interferer (i.e. K=2).

Fig. 3 displays the mean UT vs. the number of transmit-receive pairs (K) when using η=1 or η=0.5, antenna configuration 1×1 or 2×2, and SNR=20dB. It can be observed that even if we focus on minimization of the MSE, maximization of the rate is possible when using improper Gaussian signaling, either with interference management or not. Similarly as it was observed in terms of MSE in Fig. 2, the gains in terms of UT provided by the use of improper Gaussian signaling are larger for η=1 than for η=0.5, and they are also more relevant in the 1×1 case than in the 2×2 case. For K=2 it is enough, in terms of UT, applying 'improper no-IM'; IM might not be needed, as the use of improper Gaussian signaling already introduces the additional dimension needed to cancel the single potential interferer. Fig. 3 a) shows that using improper Gaussian signaling allows an additional pair of nodes included in the scenario to maintain the per-user rate while increasing the number of served UEs (proportional to K).

V. CONCLUSIONS

In this article decentralized transmit and receive beamformers optimizations with improper Gaussian signaling have been proposed and analyzed for MIMO-IC. WLP is adopted at transmission and WLE is applied at reception, which allows generating and estimating improper Gaussian signals. In order to manage the interference present in the network, a decentralized technique for coordinated WLP design that exploits UL-DL propagation channel reciprocity is derived based on MMSE criterion. The UL received signal is employed as a way of reporting the impact of measured interference by UEs served by neighboring BSs. The proposed decentralized approach provides gains in terms of MSE and user throughput as compared to conventional proper interference management schemes in highly interfered scenarios. The relative gains are observed in the following situations: 1) when the number of transmit/receive antennas is low, 2) when the level of interference is high, or 3) at high SNR regimes. If the interference level is not significant or diversity is provided by multiple antennas at transmitters and receivers, the proposed decentralized approach leads to a proper-based solution.

VI. APPENDIX – PROOF OF PROPOSITION 1

Derivation of transmit filters: The optimal transmit filters, when keeping the remaining sets of variables fixed, are obtained from the derivatives of the Lagrangian function of problem (P_0) in (8) with respect to $\mathbf{T}_k^{1*}$ and $\mathbf{T}_k^{2*}$. Let us denote by λ_k the non-negative dual variable associated to the k-th power constraint in (8). The Lagrangian function (L) is [16]:

$$L = \sum_{k=1}^{K} \mathrm{Tr}\,(\mathbf{E}_k) + \sum_{k=1}^{K} \lambda_k (\mathrm{Tr}\,(\mathbf{T}_k^1 \mathbf{T}_k^{1H} + \mathbf{T}_k^2 \mathbf{T}_k^{2H}) - P_k^{DL}). \quad (22)$$

By using the parameters defined in (9), the derivatives result:

$$\begin{aligned} \nabla_{\mathbf{T}_k^{1*}} L &= -\mathbf{H}_k^{kH} \mathbf{R}_k^1 + (\mathbf{A}_k + \lambda_k \mathbf{I}) \mathbf{T}_k^1 + \mathbf{B}_k \mathbf{T}_k^{2*} = \mathbf{0} \\ \nabla_{\mathbf{T}_k^{2*}} L &= -\mathbf{H}_k^{kH} \mathbf{R}_k^{2*} + (\mathbf{A}_k + \lambda_k \mathbf{I}) \mathbf{T}_k^2 + \mathbf{B}_k \mathbf{T}_k^{1*} = \mathbf{0} \end{aligned}, \quad (23)$$

and, from (23), linear transmit filters can be obtained:

$$\begin{aligned} \mathbf{T}_k^1 &= (\mathbf{A}_k + \lambda_k \mathbf{I})^{-1} (\mathbf{H}_k^{kH} \mathbf{R}_k^1 - \mathbf{B}_k \mathbf{T}_k^{2*}) \\ \mathbf{T}_k^2 &= (\mathbf{A}_k + \lambda_k \mathbf{I})^{-1} (\mathbf{H}_k^{kH} \mathbf{R}_k^{2*} - \mathbf{B}_k \mathbf{T}_k^{1*}) \end{aligned}. \quad (24)$$

Further, transmit filters in (24) can be decoupled by applying the substitution method and expressions in (11) are derived.

Derivation of receive filters: The optimal receive filters, when keeping the remaining sets of variables fixed, are obtained from the derivatives of the Lagrangian function of problem in (8) (i.e. L in (22)) as follows:

$$\begin{aligned} \nabla_{\mathbf{R}_k^{1*}} L &= -\mathbf{H}_k^k \mathbf{T}_k^1 + \mathbf{C}_{\mathbf{y}_k} \mathbf{R}_k^1 + \tilde{\mathbf{C}}_{\mathbf{y}_k} \mathbf{R}_k^2 = \mathbf{0} \\ \nabla_{\mathbf{R}_k^{2*}} L &= -\mathbf{H}_k^{k*} \mathbf{T}_k^{2*} + \mathbf{C}_{\mathbf{y}_k}^* \mathbf{R}_k^2 + \tilde{\mathbf{C}}_{\mathbf{y}_k}^* \mathbf{R}_k^1 = \mathbf{0} \end{aligned}. \quad (25)$$

From (25), receive filters can be uncoupled and expressions in (11) are obtained. In case the k-th user is constrained to use LE (i.e. $\mathbf{R}_k^2 = \mathbf{0}$ in (4)), the solution for the linear receive filter is derived from the first equation in (25): $\mathbf{R}_k^1 = \mathbf{C}_{\mathbf{y}_k}^{-1} \mathbf{H}_k^k \mathbf{T}_k^1$.

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