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Singular Detection in Noncoherent Communications

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***Abstract*—This paper proposes a general analysis of codeword detection in noncoherent communications. Motivated by the existence of error floors in various regimes, fundamental characteristics of signal design are investigated. In particular, the necessary and sufficient conditions for asymptotically singular detection (*i.e.* error-free in the limit) are derived from classical results in detection theory. By leveraging tools from linear algebra and the theory of Hilbert spaces, we are able to characterize asymptotic singularity in two main scenarios: the large array and high SNR regimes. The results generalize previous works and extend the notion of unique identification, as well as recontextualize the geometry of Grassmannian constellations from an alternative perspective.**

***Index Terms*—Noncoherent communications, singular detection, unique identification, massive MIMO, high SNR regime.**

I. Introduction

Noncoherent detection has been a part of cellular wireless communications since their inception [1, App. D]. First and second generation systems were defined by device manufacturing limitations, which constrained communication schemes to employ modulations that did not require instantaneous channel state information (CSI) [2]. With technological advancements came a rise in demand for higher data-rates, making spectral resources more valuable. Therefore, digital systems based on coherent detection became the norm in subsequent generations (*i.e.* 3G to 5G), due to their improved spectral efficiency.

New applications emerging within the fifth and succeeding generations showcase novel technical bottlenecks beyond spectrum scarcity. Several of these barriers are related to the acquisition of reliable instantaneous CSI, especially when employing large numbers of antennas, in high mobility scenarios or under low latency requirements [3]. All these challenges have rekindled an interest in noncoherent solutions for next generation communications.

The present work studies the error performance of noncoherent systems from the perspective of detection theory. Motivated by the existence of error floors in various communication settings [4], [5], we analyze which signal properties allow for an *asymptotically singular detection (ASD)*, *i.e.* asymptotically error-free. Understanding if a configuration will display a fundamental error floor provides valuable insights on the achievable gains obtained by pouring more resources into a system. In particular, we explore the following scenarios:

- **Large array regime** (no. receiver antennas $\to \infty$). It sheds light onto the performance improvements brought by massive arrays (such as the emergence of *channel hardening* [4]), against the infrastructure costs they entail.
- **High signal-to-noise ratio (SNR) regime** ($\mathrm{SNR} \to \infty$). Likewise, this analysis is of particular interest to assess the gains attainable by increasing the transmitted power, in view of energy efficiency and power-limited systems [3].

The main goal behind this work is to establish necessary and sufficient conditions for an alphabet to yield ASD under the presented regimes. This allows to understand fundamental limitations of noncoherent systems and unveils integral aspects of codeword design. Therefore, without sacrificing interpretability, we have considered a signal framework that encapsulates a variety of scenarios of interest, both well-established [6, Sec. 3.6.1] and state-of-the-art [7]. The main results are summarized next. In Section III, we take some classical ideas from detection theory [8] and adapt them for the large array regime (*Proposition 1*), with which we determine novel insightful requirements for ASD in such scenario. Conversely, in Section IV we derive the necessary and sufficient conditions for ASD in the high SNR regime (*Proposition 2*), which yield a powerful geometric interpretation on the problem. These results are a refinement of Theorems 1 and 2 from [5], and generalize them for any configuration of transmitting and receiving antennas and codeword length.

The notation used throughout the text is defined next. Vectors and matrices: boldface lowercase and uppercase. Transpose and conjugate transpose: $\cdot^{\mathrm{T}}$, $\cdot^{\mathrm{H}}$. Trace: $\mathrm{Tr}\{\cdot\}$. Minimum and maximum eigenvalues: $\sigma_{\min}(\cdot)$ and $\sigma_{\max}(\cdot)$. Entry (r,c) of a matrix: $[\mathbf{A}]_{r,c}$. Column space: $\mathcal{C}(\cdot)$. Column-wise vectorization: $\mathrm{vec}(\cdot)$. Kronecker product: $\otimes$. Matrix determinant: $|\mathbf{A}|$. Euclidean, Frobenius and weighted norms: $\|\mathbf{a}\|$, $\|\mathbf{X}\|_{\mathrm{F}}$, $\|\mathbf{X}\|_{\mathbf{A}} \triangleq \sqrt{\mathrm{Tr}\{\mathbf{X}^{\mathrm{H}}\mathbf{A}^{-1}\mathbf{X}\}}$. Empty set: $\{\emptyset\}$. Direct sum: $\oplus$. Random variables: sans serif font. Expectation: $\mathbb{E}[\cdot]$. Circularly symmetric complex normal vector: $\mathsf{a} \sim \mathcal{CN}(\mathbf{m}, \mathbf{C})$.

II. Preliminary notions

A. Signal model

Consider a MIMO point-to-point system, in which transmitter and receiver are equipped with N_{t} and N_{r} antennas, respectively. The channel is assumed frequency flat with a coherence time K. This translates into a channel matrix $\mathsf{H} \in \mathbb{C}^{N_{\mathrm{t}} \times N_{\mathrm{r}}}$ that remains constant for K channel uses (*i.e.* block flat fading). During each block, the transmitter sends an equiprobable codeword $\mathbf{S} \in \mathbb{C}^{K \times N_{\mathrm{t}}}$ selected from a finite alphabet $\mathcal{S}$ of size M. We assume an average power constraint of $\frac{1}{K}\mathbb{E}_{\mathsf{S}}[\|\mathsf{S}\|_{\mathrm{F}}^2] \triangleq 1$.

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The signal at the receiver is expressed as a time-space matrix, using a complex baseband representation:

$$\mathbf{Y} = \mathbf{X}\mathbf{H} + \mathbf{Z} \in \mathbb{C}^{K \times N_\mathrm{r}}, \tag{1}$$

where $\mathbf{X} \triangleq \sqrt{\mathrm{P}_\mathbf{X}} \cdot \mathbf{S}$ is the transmitted signal with average power $\mathrm{P}_\mathbf{X}$, and $\mathbf{Z}$ is an independent additive Gaussian noise component. The average channel and noise power is normalized as

$$\mathbb{E}_\mathbf{H}[\|\mathbf{H}\|_\mathrm{F}^2] \triangleq 1, \quad \tfrac{1}{K}\,\mathbb{E}_\mathbf{Z}[\|\mathbf{Z}\|_\mathrm{F}^2] \triangleq \mathrm{P}_\mathbf{Z}, \tag{2}$$

with which we define the SNR at the receiver:

$$\mathrm{SNR} \triangleq \frac{\mathbb{E}_{\mathbf{H},\mathbf{X}}[\|\mathbf{X}\mathbf{H}\|_\mathrm{F}^2]}{\mathbb{E}_\mathbf{Z}[\|\mathbf{Z}\|_\mathrm{F}^2]} = \frac{\mathrm{P}_\mathbf{X}}{\mathrm{P}_\mathbf{Z}} \cdot \frac{\mathbb{E}_{\mathbf{H},\mathbf{S}}[\|\mathbf{S}\mathbf{H}\|_\mathrm{F}^2]}{K}. \tag{3}$$

Under this model, the received signal will be finite-energy regardless of N_r, which is relevant in the study of the large array regime in Section III. Indeed, from the power normalizations in (2), the average received energy when transmitting $\mathbf{S}$ is

$$\mathbb{E}_{\mathbf{Y}|\mathbf{S}}[\|\mathbf{Y}\|_\mathrm{F}^2] = \mathrm{P}_\mathbf{X}\,\mathbb{E}_\mathbf{H}[\|\mathbf{S}\mathbf{H}\|_\mathrm{F}^2] + \mathrm{P}_\mathbf{Z}\,K. \tag{4}$$

Applying [9, Fact 10.14.22] and the maximum transmitted codeword energy,

$$\mathbb{E}_{\mathbf{Y}|\mathbf{S}}[\|\mathbf{Y}\|_\mathrm{F}^2] \leq \mathrm{P}_\mathbf{X}\,MK + \mathrm{P}_\mathbf{Z}\,K, \tag{5}$$

which is clearly bounded for any N_r.

The distributions of both $\mathbf{H}$ and $\mathbf{Z}$ are assumed to be known by the receiver but not their realizations[1]. To characterize them statistically, it is convenient to vectorize the received signal matrix column-wise as follows:

$$\begin{aligned}\widetilde{\mathbf{y}} &\triangleq \mathrm{vec}(\mathbf{Y}) = (\mathbf{I}_{N_\mathrm{r}} \otimes \mathbf{X})\,\mathrm{vec}(\mathbf{H}) + \mathrm{vec}(\mathbf{Z}) \\ &\triangleq \widetilde{\mathbf{X}}\widetilde{\mathbf{h}} + \widetilde{\mathbf{z}} \in \mathbb{C}^{KN_\mathrm{r}}.\end{aligned} \tag{6}$$

We define $\widetilde{\mathbf{S}} \triangleq \mathbf{I}_{N_\mathrm{r}} \otimes \mathbf{S}$ in the same manner. Assuming correlated Rayleigh fading, the vectorized channel matrix is distributed as $\widetilde{\mathbf{h}} \sim \mathcal{CN}(\mathbf{0}_{N_\mathrm{t}N_\mathrm{r}}, \mathbf{C}_{\widetilde{\mathbf{h}}})$. Similarly, the noise is distributed as $\widetilde{\mathbf{z}} \sim \mathcal{CN}(\mathbf{0}_{KN_\mathrm{r}}, \mathbf{C}_{\widetilde{\mathbf{z}}})$ and its covariance matrix is assumed full-rank, without loss of generality.

B. Error probability of ML detection

An important metric to consider when designing a digital communication system is the error probability of codeword detection. When dealing with an equiprobable alphabet, the receiver that minimizes it is the *maximum likelihood (ML) detector* [10, Thm. 21.3.3]. It is derived from the likelihood function of the received signal (6), conditioned to a transmitted codeword $\mathbf{S}_i$ (*i.e.* $\mathbf{X}_i$) and a channel realization $\mathbf{H}$:

$$\mathrm{f}_{\mathbf{Y}|\mathbf{S}_i,\mathbf{H}}(\widetilde{\mathbf{y}}) = \tfrac{1}{\pi^{KN_\mathrm{r}}|\mathbf{C}_{\widetilde{\mathbf{z}}}|}\, e^{-(\widetilde{\mathbf{y}} - \widetilde{\mathbf{X}}_i\widetilde{\mathbf{h}})^\mathrm{H}\mathbf{C}_{\widetilde{\mathbf{z}}}^{-1}(\widetilde{\mathbf{y}} - \widetilde{\mathbf{X}}_i\widetilde{\mathbf{h}})}. \tag{7}$$

Since the channel realization is unknown at the receiver, the uncertainty of $\mathbf{H}$ can be treated as a random variable (*i.e. unconditional model* [11]) and removed from the conditioning by marginalization:

$$\mathrm{f}_{\mathbf{Y}|\mathbf{S}_i}(\widetilde{\mathbf{y}}) = \mathbb{E}_\mathbf{H}[\mathrm{f}_{\mathbf{Y}|\mathbf{S}_i,\mathbf{H}}(\widetilde{\mathbf{y}})] = \tfrac{1}{\pi^{KN_\mathrm{r}}|\mathbf{C}_i|}\, e^{-\widetilde{\mathbf{y}}^\mathrm{H}\mathbf{C}_i^{-1}\widetilde{\mathbf{y}}}, \tag{8}$$

where $\mathbf{C}_i \triangleq \widetilde{\mathbf{X}}_i\mathbf{C}_{\widetilde{\mathbf{h}}}\widetilde{\mathbf{X}}_i^\mathrm{H} + \mathbf{C}_{\widetilde{\mathbf{z}}}$ is the covariance matrix of the received signal conditioned to $\mathbf{S}_i$. With (8) we can obtain the ML detector by maximizing it over all possible codewords from $\mathcal{S}$: $\widehat{\mathbf{S}}_\mathrm{ML} = \arg\max_{\mathbf{S}_i \in \mathcal{S}} \mathrm{f}_{\mathbf{Y}|\mathbf{S}_i}(\widetilde{\mathbf{y}})$.

[1]This is referred to as *statistical CSI at the receiver*.

C. Pairwise error probability

The probability of error of a communication system is usually analytically intractable, so various works in the literature [12] resort to the much simpler *pairwise error probability (PEP)* between two codewords. It is defined as:

$$\begin{aligned}\mathrm{P}_{a\to b} &\triangleq \Pr\{\mathrm{f}_{\mathbf{Y}|\mathbf{S}_a}(\widetilde{\mathbf{y}}) \leq \mathrm{f}_{\mathbf{Y}|\mathbf{S}_b}(\widetilde{\mathbf{y}}) \,|\, \mathbf{S} = \mathbf{S}_a\} \\ &= \Pr\{\mathrm{L}_{a,b}(\widetilde{\mathbf{y}}) \leq 0 \,|\, \mathbf{S} = \mathbf{S}_a\},\end{aligned} \tag{9}$$

where

$$\mathrm{L}_{a,b}(\widetilde{\mathbf{y}}) \triangleq \ln\frac{\mathrm{f}_{\mathbf{Y}|\mathbf{S}_a}(\widetilde{\mathbf{y}})}{\mathrm{f}_{\mathbf{Y}|\mathbf{S}_b}(\widetilde{\mathbf{y}})} = \widetilde{\mathbf{y}}^\mathrm{H}(\mathbf{C}_b^{-1} - \mathbf{C}_a^{-1})\widetilde{\mathbf{y}} - \ln\frac{|\mathbf{C}_a|}{|\mathbf{C}_b|} \tag{10}$$

is the *log-likelihood ratio (LLR)* between hypotheses a and b.

The PEPs of an alphabet are very insightful tools to analyze the performance of its design. With the maximum PEP of $\mathcal{S}$, we can bound its detection error probability P_e as [12]:

$$\max_{\mathbf{S}_a \neq \mathbf{S}_b \in \mathcal{S}} \tfrac{1}{M}\mathrm{P}_{a\to b} \leq \mathrm{P}_\mathrm{e} \leq \max_{\mathbf{S}_a \neq \mathbf{S}_b \in \mathcal{S}} (M-1)\mathrm{P}_{a\to b}. \tag{11}$$

This implies the detection error probability of $\mathcal{S}$ will vanish if and only if its maximum PEP does as well.

D. Unique identification

Definition 1. *An alphabet $\mathcal{S}$ is* uniquely identifiable *[5] if*

$$\mathbf{C}_a \neq \mathbf{C}_b \iff \mathbf{S}_a \neq \mathbf{S}_b, \quad \forall \mathbf{S}_a, \mathbf{S}_b \in \mathcal{S}. \tag{12}$$

This property is fundamental to communication systems working noncoherently, as displayed in the following lemma.

Lemma 1. *Unique identification is a necessary condition for an alphabet to be detected with arbitrarily low P_e under the model from Section II-A.*

Proof: The proof is straightforward. Having $\mathbf{C}_a = \mathbf{C}_b$ for two different codewords $\mathbf{S}_a, \mathbf{S}_b \in \mathcal{S}$ collapses their LLR, *i.e.* $\mathrm{L}_{a,b}(\widetilde{\mathbf{y}}) = 0$, $\forall \widetilde{\mathbf{y}} \in \mathbb{C}^{KN_\mathrm{r}}$. This prevents the receiver from distinguishing between them, so their associated PEP is always positively lower-bounded. Therefore, unique identification is necessary for singular detection. ■

III. Large array regime

The next result is a restatement of [8, Lemma 3], which is based on [13, Ch. 8, 9]. We consider it to establish under which criteria noncoherent detection will benefit from increasing N_r. To expound it, we introduce the *Jeffreys divergence (J-div.)*[2] [13, Ch. 8] between the distributions of $\widetilde{\mathbf{y}}|\mathbf{S}_a$ and $\widetilde{\mathbf{y}}|\mathbf{S}_b$:

$$\mathrm{J}_{N_\mathrm{r}} \triangleq \mathbb{E}_{\mathbf{Y}|\mathbf{S}_a}[\mathrm{L}_{a,b}(\widetilde{\mathbf{y}})] - \mathbb{E}_{\mathbf{Y}|\mathbf{S}_b}[\mathrm{L}_{a,b}(\widetilde{\mathbf{y}})]. \tag{13}$$

Taking this into account, the aforementioned result is displayed next.

Proposition 1 (Kailath & Weinert [8])**.** *A* necessary *and* sufficient *condition for alphabet $\mathcal{S}$ to be detected with vanishing P_e as $N_\mathrm{r} \to \infty$ under the model from Section II-A is*

$$\lim_{N_\mathrm{r}\to\infty} \mathrm{J}_{N_\mathrm{r}} = \infty, \tag{14}$$

for all $\mathbf{S}_a \neq \mathbf{S}_b$ in $\mathcal{S}$.

[2]Some authors (see [14]) define the J-div. as half of (13).

Particularizing the J-div. to our signal model yields

$$\mathrm{J}_{N_\mathrm{r}} = \mathrm{Tr}\{(\mathbf{C}_b^{-1} - \mathbf{C}_a^{-1})(\mathbf{C}_a - \mathbf{C}_b)\}, \quad (15)$$

where we have used the circularity and linearity properties of the trace. The signal normalization considered herein allows the obtained criterion to only capture nontrivial situations [8]. On the one hand, singularity will not emerge for finite N_r thanks to[3] $\mathbf{C}_\diamond$ being strictly positive-definite, due to the rank-completeness of $\mathbf{C}_{\widetilde{\mathbf{z}}}$. On the other hand, ASD will never arise as a result of trivially increasing the received power, since all the entries of $\mathbf{C}_\diamond$ are bounded. This can be shown with the Cauchy-Schwarz inequality:

$$|[\mathbf{C}_\diamond]_{r,c}| \leq \sqrt{[\mathbf{C}_\diamond]_{r,r}[\mathbf{C}_\diamond]_{c,c}} \leq \mathrm{Tr}\{\mathbf{C}_\diamond\} < \infty. \quad (16)$$

The last inequality can be stated because $\widetilde{\mathbf{y}}|\mathbf{S}_\diamond$ is finite-energy for any N_r, as proved in (5).

We may express (15) in the more insightful form

$$\mathrm{J}_{N_\mathrm{r}} = \left\|\mathbf{C}_a^{-\frac{1}{2}}(\mathbf{C}_a - \mathbf{C}_b)\mathbf{C}_b^{-\frac{1}{2}}\right\|_\mathrm{F}^2. \quad (17)$$

With it, Proposition 1 defines a notion of unique identification for increasing dimensionality: the inequality from Lemma 1 is replaced by a divergent norm, in a similar manner as how equality and strong convergence are related when dealing with Hilbert spaces [15, Sec. 2.8.1].

In [5, Thm. 1], a criterion for ASD in the large array regime was derived for a single channel use ($K = 1$) SIMO ($N_\mathrm{t} = 1$) system. Not only is Proposition 1 a more general result applicable to any configuration of K and N_t, but it also entails a clear refinement. Whereas in [5] elaborate mathematical machinery had to be deployed (*e.g. Cantelli's inequality* and convergence tests), Proposition 1 allows us to reach the conclusions presented in that particular analysis from a much more straightforward procedure. Indeed, particularizing (17) for $K = N_\mathrm{t} = 1$ yields

$$\mathrm{J}_{N_\mathrm{r}} = \Delta_{a,b}^2 \cdot \mathrm{Tr}\{\mathbf{C}_{\widetilde{\mathbf{h}}}\mathbf{C}_b^{-1}\mathbf{C}_{\widetilde{\mathbf{h}}}\mathbf{C}_a^{-1}\}, \quad (18)$$

for $\Delta_{a,b} \triangleq |x_a|^2 - |x_b|^2$. With simple manipulations we can bound the previous expression as

$$\frac{\Delta_{a,b}^2}{(|x_a|^2C+1)(|x_b|^2C+1)} \cdot \mathrm{Tr}\{\mathbf{\Gamma}^2\} \leq \mathrm{J}_{N_\mathrm{r}} \leq \Delta_{a,b}^2 \cdot \mathrm{Tr}\{\mathbf{\Gamma}^2\}, \quad (19)$$

where $\mathbf{\Gamma} \triangleq \mathbf{C}_{\widetilde{\mathbf{z}}}^{-\frac{1}{2}}\mathbf{C}_{\widetilde{\mathbf{h}}}\mathbf{C}_{\widetilde{\mathbf{z}}}^{-\frac{1}{2}}$ and $C \triangleq \sigma_{\max}(\mathbf{\Gamma}) > 0$. This allows splitting the divergence study of (18) into two simpler criteria. The first one depends on the statistics of channel and noise in the studied model, which must yield $\lim_{N_\mathrm{r}\to\infty} \mathrm{Tr}\{\mathbf{\Gamma}^2\} = \infty$. This is related to the decay rate of signal and noise spectra discussed in [16, Sec. 3]. If this condition is met, the second one depends on the design of the transmitted signal: ASD will be achieved for $|x_a|^2 \neq |x_b|^2$, so that $\Delta_{a,b}^2$ in (18) and (19) is strictly positive.

The J-div. measures how different $\mathrm{f}_{\mathbf{Y}|\mathbf{S}_a}$ and $\mathrm{f}_{\mathbf{Y}|\mathbf{S}_b}$ are, and is null if and only if $\mathbf{S}_a$ and $\mathbf{S}_b$ are not uniquely identifiable [17, Prop. 12]. Moreover, when $\widetilde{\mathbf{y}}|\mathbf{S}_a$ and $\widetilde{\mathbf{y}}|\mathbf{S}_b$ yield close distributions, the J-div. approximates the squared geodesic distance between them over the statistical manifold defined by the Fisher information metric [14]. This distance is commonly expressed in terms of the *normalized covariance matrix* [17] $\mathring{\mathbf{C}} \triangleq \mathbf{C}_b^{-\frac{1}{2}}\mathbf{C}_a\mathbf{C}_b^{-\frac{1}{2}}$. Similarly, we can use it to represent $\mathrm{J}_{N_\mathrm{r}}$:

$$\mathrm{J}_{N_\mathrm{r}} = \|\mathring{\mathbf{C}} - \mathbf{I}_{KN_\mathrm{r}}\|_{\mathring{\mathbf{C}}}^2, \quad (20)$$

which clearly measures how much $\mathring{\mathbf{C}}$ differs from $\mathbf{I}_{KN_\mathrm{r}}$. This dissimilarity (and thus the one between $\mathbf{C}_a$ and $\mathbf{C}_b$) must increase with N_r for $\mathrm{J}_{N_\mathrm{r}}$ to diverge.

On a final note, a set of equivalent conditions for ASD can be derived from (20), as acknowledged in [8, Lemma 4]. The J-div. will diverge if and only if

$$\|\mathring{\mathbf{C}} - \mathbf{I}_{KN_\mathrm{r}}\|_\mathrm{F}^2 = \|\mathbf{C}_a - \mathbf{C}_b\|_{\mathbf{C}_b}^2 \to \infty, \quad \sigma_{\min}(\mathring{\mathbf{C}}) > 0, \quad (21)$$

as $N_\mathrm{r} \to \infty$. One would intuitively expect the metric that arises for ASD in the large array regime to be $\|\mathbf{C}_a - \mathbf{C}_b\|_\mathrm{F}^2$ (*i.e.* the Euclidean distance), as an extension of Lemma 1. Remarkably, we have established in (17) that this is not the case. Even with the resemblance between (21) and the intuitive Euclidean distance, the relevant metric to assess ASD is a norm defined from hypotheses a and b [8].

IV. High SNR regime

The next results are derived for full-rank $\mathbf{C}_{\widetilde{\mathbf{h}}}$. Their extension to the rank-deficient case can be obtained by transforming the model in Section II-A onto a lower-dimensional space.

Proposition 2. *A* necessary *and* sufficient *condition for a constellation $\mathcal{S}$ to be detected with vanishing* $\mathrm{P_e}$ *as* $\mathrm{SNR} \to \infty$ *under the model from Section II-A is*

$$\mathcal{C}(\mathbf{S}_a) \neq \mathcal{C}(\mathbf{S}_b) \iff \mathbf{S}_a \neq \mathbf{S}_b, \quad \forall \mathbf{S}_a, \mathbf{S}_b \in \mathcal{S}. \quad (22)$$

Proof: Let $\gamma \triangleq \mathrm{P}_\mathbf{X} / \mathrm{P}_\mathbf{Z}$. We can restate definition (3) as $\mathrm{SNR} = \gamma \cdot \mathbb{E}_\mathbf{S}[\mathrm{Tr}\{\widetilde{\mathbf{S}}^\mathrm{H}\widetilde{\mathbf{S}}\mathbf{C}_{\widetilde{\mathbf{h}}}\}]/K$. Using [9, Prop. 10.4.13], we can bound the SNR as

$$\gamma N_\mathrm{r} \cdot \sigma_{\min}(\mathbf{C}_{\widetilde{\mathbf{h}}}) \leq \mathrm{SNR} \leq \gamma N_\mathrm{r} \cdot \sigma_{\max}(\mathbf{C}_{\widetilde{\mathbf{h}}}). \quad (23)$$

Therefore, $\mathrm{SNR} \to \infty \iff \gamma \to \infty$.

The PEP in (9) can be expressed in integral form:

$$\mathrm{P}_{a\to b} = \Pr\{\widetilde{\mathbf{y}} \in \mathcal{P} | \mathbf{S} = \mathbf{S}_a\} = \int_\mathcal{P} \mathrm{f}_{\mathbf{Y}|\mathbf{S}_a}(\widetilde{\mathbf{y}})\mathrm{d}\widetilde{\mathbf{y}}, \quad (24)$$

as stated in [10, Sec. 20.5]. The region of integration is

$$\mathcal{P} \triangleq \{\widetilde{\mathbf{y}} \in \mathbb{C}^{KN_\mathrm{r}} : \mathrm{L}_{a,b}(\widetilde{\mathbf{y}}) \leq 0\}. \quad (25)$$

We are interested in the behavior of (24) as $\gamma \to \infty$. As a first step, we define the normalized covariance matrix $\overline{\mathbf{C}}_{\widetilde{\mathbf{z}}} \triangleq \frac{1}{\mathrm{P}_\mathbf{Z}}\mathbf{C}_{\widetilde{\mathbf{z}}}$, with which we whiten the received signal: $\mathbf{r} \triangleq (\mathrm{P}_\mathbf{X}\,\overline{\mathbf{C}}_{\widetilde{\mathbf{z}}})^{-\frac{1}{2}}\widetilde{\mathbf{y}}$. The PEP integral in terms of $\mathbf{r}$ becomes [10, Lemma 17.4.6]:

$$\mathrm{P}_{a\to b} = \int_{\mathcal{P}'} \frac{1}{\pi^{KN_\mathrm{r}}|\mathbf{D}_a(\gamma)|} e^{-\mathbf{r}^\mathrm{H}\mathbf{D}_a^{-1}(\gamma)\mathbf{r}}\mathrm{d}\mathbf{r}, \quad (26)$$

where we have defined

$$\mathbf{D}_\diamond(\gamma) \triangleq \mathbf{\Xi}_\diamond + \frac{1}{\gamma}\mathbf{I}_{KN_\mathrm{r}}, \quad \mathbf{\Xi}_\diamond \triangleq \overline{\mathbf{C}}_{\widetilde{\mathbf{z}}}^{-\frac{1}{2}}\widetilde{\mathbf{S}}_\diamond\mathbf{C}_{\widetilde{\mathbf{h}}}\widetilde{\mathbf{S}}_\diamond^\mathrm{H}\overline{\mathbf{C}}_{\widetilde{\mathbf{z}}}^{-\frac{1}{2}}. \quad (27)$$

The new region of integration is

$$\mathcal{P}' = \left\{\mathbf{r} \in \mathbb{C}^{KN_\mathrm{r}} : \|\mathbf{r}\|_{\mathbf{D}_b(\gamma)}^2 - \|\mathbf{r}\|_{\mathbf{D}_a(\gamma)}^2 \leq \ln\frac{|\mathbf{D}_a(\gamma)|}{|\mathbf{D}_b(\gamma)|}\right\}. \quad (28)$$

[3] The symbol "$\diamond$" will be used throughout the text as a placeholder for both "a" and "b" indistinctly.

To analyze the limit of the PEP when $\gamma \to \infty$, it is convenient to restate (26) in terms of the following orthogonal decomposition: $\mathbb{C}^{KN_\mathrm{r}} \triangleq \mathrm{V}_\diamond \oplus \mathrm{V}_\diamond^\perp$. The $N_\diamond$-dimensional signal subspace $\mathrm{V}_\diamond$ is spanned by $\mathbf{U}_\diamond \in \mathbb{C}^{KN_\mathrm{r} \times N_\diamond}$, which contains the eigenvectors of $\mathbf{\Xi}_\diamond$. Its non-zero eigenvalues are positive and can be grouped in a diagonal matrix $\mathbf{\Lambda}_\diamond \in \mathbb{R}^{N_\diamond \times N_\diamond}$. Similarly, $\mathrm{V}_\diamond^\perp$ is the orthogonal complement of $\mathrm{V}_\diamond$, which corresponds to the noise subspace and has dimension $N_\diamond^\perp \triangleq KN_\mathrm{r} - N_\diamond$. It is spanned by $\mathbf{U}_\diamond^\perp \in \mathbb{C}^{KN_\mathrm{r} \times N_\diamond^\perp}$.

With the previous definitions, (27) becomes

$$\mathbf{D}_\diamond(\gamma) \triangleq \mathbf{U}_\diamond\big(\mathbf{\Lambda}_\diamond + \tfrac{1}{\gamma}\mathbf{I}_{N_\diamond}\big)\mathbf{U}_\diamond^\mathrm{H} + \tfrac{1}{\gamma}\mathbf{U}_\diamond^\perp\mathbf{U}_\diamond^{\perp\mathrm{H}}. \tag{29}$$

Every vector in $\mathbb{C}^{KN_\mathrm{r}}$ can be decomposed as $\mathbf{r} \triangleq \mathbf{r}_\diamond + \mathbf{r}_\diamond^\perp$, where $\mathbf{r}_\diamond \in \mathrm{V}_\diamond$ and $\mathbf{r}_\diamond^\perp \in \mathrm{V}_\diamond^\perp$. Each component is obtained by orthogonally projecting $\mathbf{r}$ onto the corresponding subspace, using projection matrices $\mathbf{P}_\diamond \triangleq \mathbf{U}_\diamond\mathbf{U}_\diamond^\mathrm{H}$ and $\mathbf{P}_\diamond^\perp \triangleq \mathbf{U}_\diamond^\perp\mathbf{U}_\diamond^{\perp\mathrm{H}}$, respectively. The PEP integral in terms of $\mathrm{V}_\diamond$ and $\mathrm{V}_\diamond^\perp$ is

$$\mathrm{P}_{a\to b} = \int_{\mathcal{P}'} \frac{e^{-\mathbf{r}_a^\mathrm{H}\mathbf{U}_a(\mathbf{\Lambda}_a + \frac{1}{\gamma}\mathbf{I}_{N_a})^{-1}\mathbf{U}_a^\mathrm{H}\mathbf{r}_a}}{\pi^{N_a}|\mathbf{\Lambda}_a + \frac{1}{\gamma}\mathbf{I}_{N_a}|} \frac{e^{-\gamma\|\mathbf{r}_a^\perp\|^2}}{(\pi/\gamma)^{N_a^\perp}} \mathrm{d}\mathbf{r}_a \mathrm{d}\mathbf{r}_a^\perp. \tag{30}$$

To obtain its limit as $\gamma \to \infty$, we apply a simple change of variable $\mathbf{t}_a^\perp \triangleq \sqrt{\gamma} \cdot \mathbf{r}_a^\perp$:

$$\mathrm{P}_{a\to b} = \int_{\mathcal{P}'} \frac{e^{-\mathbf{r}_a^\mathrm{H}\mathbf{U}_a(\mathbf{\Lambda}_a + \frac{1}{\gamma}\mathbf{I}_{N_a})^{-1}\mathbf{U}_a^\mathrm{H}\mathbf{r}_a}}{\pi^{N_a}|\mathbf{\Lambda}_a + \frac{1}{\gamma}\mathbf{I}_{N_a}|} \frac{e^{-\|\mathbf{t}_a^\perp\|^2}}{\pi^{N_a^\perp}} \mathrm{d}\mathbf{r}_a \mathrm{d}\mathbf{t}_a^\perp. \tag{31}$$

The integrand of this expression converges pointwise to

$$\tfrac{1}{\pi^{N_a}|\mathbf{\Lambda}_a|} e^{-\mathbf{r}_a^\mathrm{H}\mathbf{U}_a\mathbf{\Lambda}_a^{-1}\mathbf{U}_a^\mathrm{H}\mathbf{r}_a} \cdot \tfrac{1}{\pi^{N_a^\perp}} e^{-\|\mathbf{t}_a^\perp\|^2} \tag{32}$$

as $\gamma \to \infty$. This allows the use of Lebesgue's Dominated Convergence Theorem [18, Thm. 11.3.13] onto the limit of (31):

$$\lim_{\gamma\to\infty} \mathrm{P}_{a\to b} = \int_{\mathcal{P}'_\infty} \frac{e^{-\mathbf{r}_a^\mathrm{H}\mathbf{U}_a\mathbf{\Lambda}_a^{-1}\mathbf{U}_a^\mathrm{H}\mathbf{r}_a}}{\pi^{N_a}|\mathbf{\Lambda}_a|} \frac{e^{-\|\mathbf{t}_a^\perp\|^2}}{\pi^{N_a^\perp}} \mathrm{d}\mathbf{r}_a \mathrm{d}\mathbf{t}_a^\perp, \tag{33}$$

where $\mathcal{P}'_\infty \triangleq \lim_{\gamma\to\infty} \mathcal{P}'$. This region of integration has been developed in (35), where we have defined

$$\kappa_{a,b}(\gamma) \triangleq \ln|\mathbf{\Lambda}_a| - \ln|\mathbf{\Lambda}_b| + (N_a - N_b)\ln\gamma. \tag{34}$$

The high SNR limit of the PEP in (33) involves integrating a non-negative density function over $\mathcal{P}'_\infty$. Therefore, to evaluate its behavior, we must analyze the structure of such region under every configuration of V_a and V_b. We can immediately notice that the left-hand side (LHS) of (35) will remain bounded as $\gamma \to \infty$ when $\mathbf{P}_b^\perp\mathbf{r}_a = \mathbf{0}$, *i.e.* for all $\mathbf{r} \in \mathbb{C}^{KN_\mathrm{r}}$ such that their orthogonal projection onto V_a belongs to V_b. This will only occur when $\mathrm{V}_a \subseteq \mathrm{V}_b$. Otherwise, it will diverge with $O(\gamma)$.

Based on these observations, we can study three separate scenarios:

1) $\mathrm{V}_a \nsubseteq \mathrm{V}_b$: The LHS of (35) diverges to ∞ as $\gamma \to \infty$ for every $\mathbf{r} \in \mathbb{C}^{KN_\mathrm{r}}$. The right-hand side can display different behaviors depending on N_a and N_b:
   - $N_a < N_b$: It diverges to $-\infty$.
   - $N_a = N_b$: It remains bounded.
   - $N_a > N_b$: It diverges to ∞ with $O(\ln\gamma)$, *i.e.* slower than the LHS.

   There is no element of $\mathbb{C}^{KN_\mathrm{r}}$ that belongs to $\mathcal{P}'_\infty$ in any of the three possible cases. Therefore, $\mathcal{P}'_\infty = \{\emptyset\}$ and the PEP will vanish.
2) $\mathrm{V}_a \subset \mathrm{V}_b$: The LHS of (35) remains bounded as $\gamma \to \infty$. Since $N_a < N_b$, $\lim_{\gamma\to\infty}\kappa_{a,b}(\gamma) = -\infty$, so no element of $\mathbb{C}^{KN_\mathrm{r}}$ belongs to $\mathcal{P}'_\infty$ and the PEP will vanish.
3) $\mathrm{V}_a \equiv \mathrm{V}_b$: Both sides of (35) remain bounded, since $N_a = N_b$. The region of integration reduces to

$$\mathcal{P}'_\infty = \big\{\mathbf{r} \in \mathbb{C}^{KN_\mathrm{r}} : \|\mathbf{U}_b^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_b} - \|\mathbf{U}_a^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_a} \\ \leq \ln|\mathbf{\Lambda}_a| - \ln|\mathbf{\Lambda}_b|\big\}, \tag{36}$$

   which is delimited by a quadric in V_a and does not depend on $\mathbf{t}_a^\perp$. The asymptotic PEP in (33) is thus

$$\lim_{\gamma\to\infty} \mathrm{P}_{a\to b} = \int_{\mathcal{P}'_\infty} \frac{e^{-\|\mathbf{U}_a^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_a}}}{\pi^{N_a}|\mathbf{\Lambda}_a|} \mathrm{d}\mathbf{r}_a \int_{\mathrm{V}_a^\perp} \frac{e^{-\|\mathbf{t}_a^\perp\|^2}}{\pi^{N_a^\perp}} \mathrm{d}\mathbf{t}_a^\perp \\ = \int_{\mathcal{P}'_\infty} \frac{e^{-\|\mathbf{U}_a^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_a}}}{\pi^{N_a}|\mathbf{\Lambda}_a|} \mathrm{d}\mathbf{r}_a > 0. \tag{37}$$

   Therefore, when $\mathrm{V}_a \equiv \mathrm{V}_b$, the PEP is positively lower-bounded.

As proved in the previous analysis, the only configuration in which the PEP does not vanish as $\gamma \to \infty$ is when $\mathrm{V}_a \equiv \mathrm{V}_b$. Hence, error-free detection can be asymptotically achieved in the high SNR regime by using an alphabet $\mathcal{S}$ such that

$$\mathcal{C}(\mathbf{\Xi}_a) \neq \mathcal{C}(\mathbf{\Xi}_b) \iff \mathbf{S}_a \neq \mathbf{S}_b, \quad \forall \mathbf{S}_a, \mathbf{S}_b \in \mathcal{S}. \tag{38}$$

From [9, Thm. 3.5.3], we know that $\mathcal{C}(\mathbf{\Xi}_\diamond) = \mathcal{C}(\overline{\mathbf{C}}_{\tilde{\mathsf{z}}}\widetilde{\mathbf{S}}_\diamond\mathbf{C}_{\tilde{\mathsf{h}}})$. By definition, this column space is stated as

$$\mathcal{C}(\mathbf{\Xi}_\diamond) \triangleq \big\{\mathbf{u} \in \mathcal{C}\big(\overline{\mathbf{C}}_{\tilde{\mathsf{z}}}\big) : \widetilde{\mathbf{S}}_\diamond\mathbf{v} = \mathbf{u}, \forall\mathbf{v} \in \mathcal{C}\big(\mathbf{C}_{\tilde{\mathsf{h}}}\big)\big\}. \tag{39}$$

Since both $\overline{\mathbf{C}}_{\tilde{\mathsf{z}}}$ and $\mathbf{C}_{\tilde{\mathsf{h}}}$ are rank-complete, their column spaces are $\mathcal{C}(\overline{\mathbf{C}}_{\tilde{\mathsf{z}}}) = \mathbb{C}^{KN_\mathrm{r}}$ and $\mathcal{C}(\mathbf{C}_{\tilde{\mathsf{h}}}) = \mathbb{C}^{N_\mathrm{t}N_\mathrm{r}}$. Therefore,

$$\mathcal{C}(\mathbf{\Xi}_\diamond) = \big\{\mathbf{u} : \widetilde{\mathbf{S}}_\diamond\mathbf{v} = \mathbf{u}\big\} \equiv \mathcal{C}\big(\widetilde{\mathbf{S}}_\diamond\big). \tag{40}$$

Considering these derivations, and by construction of $\widetilde{\mathbf{S}}_\diamond$, it is straightforward to see that

$$\mathcal{C}(\mathbf{\Xi}_a) \neq \mathcal{C}(\mathbf{\Xi}_b) \iff \mathcal{C}(\mathbf{S}_a) \neq \mathcal{C}(\mathbf{S}_b). \tag{41}$$

This completes the proof. ■

A. Comments on Proposition 2

The previous result states that an alphabet will allow for ASD in the high SNR regime if and only if each codeword spans a different subspace. In broad strokes, this implies that the spectrum shape of $\mathbf{C}_\diamond$ becomes irrelevant at high SNR

$$\mathcal{P}'_\infty = \Big\{\mathbf{r} \in \mathbb{C}^{KN_\mathrm{r}} : \|\mathbf{U}_b^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_b} - \|\mathbf{U}_a^\mathrm{H}\mathbf{r}_a\|^2_{\mathbf{\Lambda}_a} + \|\mathbf{U}_b^{\perp\mathrm{H}}\mathbf{t}_a^\perp\|^2 - \|\mathbf{t}_a^\perp\|^2 + \lim_{\gamma\to\infty}\gamma\|\mathbf{U}_b^{\perp\mathrm{H}}\mathbf{r}_a\|^2 + 2\sqrt{\gamma}\Re\big\{\mathbf{r}_a^\mathrm{H}\mathbf{P}_b^\perp\mathbf{t}_a^\perp\big\} \leq \lim_{\gamma\to\infty}\kappa_{a,b}(\gamma)\Big\} \tag{35}$$

and only the signal geometry remains[4]. In a sense, the difference in covariance matrices required for unique identification (Lemma 1) is replaced by a difference in signal projection matrices.

Proposition 2 is a generalization of [5, Thm. 2] for any K and N_t. In that work, energy-based constellations were investigated for $N_\mathrm{t} = K = 1$. The existence of a high-SNR error floor was proved in such scenario if and only if $M > 2$. The same conclusion can be reached from Proposition 2:

- When $M = 2$, the transmitted symbols are $x_0 = 0$ and $x_1 > 0$. The null symbol spans a subspace of dimension 0, while x_1 spans one of maximum dimension at the receiver. By Proposition 2, this guarantees ASD.
- On the contrary, when $M > 2$, at least a pair of symbols will have non-null energy, both spanning the full available space at the receiver. Since their PEP will be positively lower-bounded, the detection of such constellation will have an error floor at high SNR.

Another noteworthy implication of this result is how it relates to *unitary space-time modulation (USTM)* [20]. It is known that, under isotropic Rayleigh fading, it achieves a vanishing gap from the channel capacity in the high SNR regime for some configurations of K, N_t and N_r [21]. Moreover, it reaches the optimal degrees of freedom of the channel in various other cases [22]. For $K \geq N_\mathrm{t}$, each codeword in USTM is constructed from a truncated unitary matrix [12] (*i.e.* $\mathbf{S}_i^\mathrm{H}\mathbf{S}_i = \mathbf{I}_{N_\mathrm{t}}$ and $\mathbf{S}_i\mathbf{S}_i^\mathrm{H} = \mathbf{P}_i$) and corresponds to a different point in the *Grassmann manifold* $\mathcal{G}(N_\mathrm{t}, \mathbb{C}^K)$ [21]. Therefore, the columns of each element in a *Grassmannian constellation (GC)* span a different N_t-dimensional subspace in $\mathbb{C}^K$.

It is clear that these codewords satisfy condition (22) and yield ASD in the high SNR regime. However, Proposition 2 hints at a more general class of constellations, of which GCs are a special case such that each codeword spans a subspace of the same dimension. A *subspace-based codebook* that would relax this constraint could be constructed as

$$\mathcal{S}_\text{subspace} \triangleq \bigcup_{n=0}^{N_\mathrm{t}} \mathcal{S}_n, \tag{42}$$

where $\mathcal{S}_n$ is a GC corresponding to $\mathcal{G}(n, \mathbb{C}^K)$. Notice that $\mathcal{G}(0, \mathbb{C}^K)$ will contain a single element at most (*i.e.* the null codeword $\mathbf{0}_{K\times N_\mathrm{t}}$). The same occurs with $\mathcal{G}(N_\mathrm{t}, \mathbb{C}^K)$ when $N_\mathrm{t} = K$, in which the only possible codeword spans the full available K-dimensional space.

V. Concluding remarks

This work has explored essential aspects of noncoherent detection, by translating well-established results from detection theory to the context of noncoherent MIMO communications. In particular, necessary and sufficient conditions for ASD have been obtained in the large array and high SNR regimes. These results have provided new insights onto relevant metrics and signal structures under each scenario.

On the one hand, Proposition 1 establishes a principle for testing whether the error probability of a system model suffers a fundamental limitation that cannot be overcome by increasing the number of receiving elements. A variety of channel profiles and codewords can be analyzed under this criterion. On the other hand, Proposition 2 reveals an umbrella class of alphabets that achieve ASD at high SNR. The presented framework leads to further research, for example in the direction of deriving benchmarks and/or alternatives to GCs, exhibiting potentially improved performance and detection complexity.

[4]This phenomenon is related to the *estimator-correlator*, whose coefficients flatten as the SNR increases [19, Sec. 5.3].

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