

ON THE UNILATERAL APPROXIMATION CONDITION WITH LINEAR ARRAYS IN NEAR-FIELD NLOS PROPAGATION

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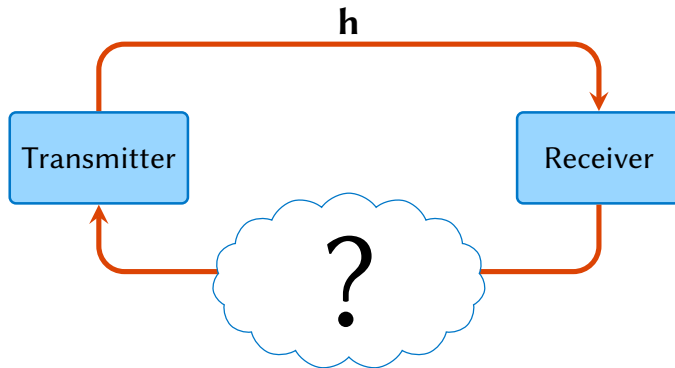
Barcelona, October 2026



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Motivation



- What is the impact of receiving antennas on transmitting ones?
- In the far field: negligible (unilateral approximation).
- Large arrays & high frequency $\rightarrow$ Near field.

Outline

- 1 System Model
- 2 Multiport Communication Theory
- 3 Asymptotic Analysis
- 4 Numerical Results
- 5 Additional Scenarios
- 6 Conclusions

System Model

- Single-antenna single user.
- N side-by-side Hertzian dipoles:

$$Z_{\parallel}(r) = \frac{3}{2} R_r j e^{-jkr} \left(\frac{1}{kr} - \frac{j}{(kr)^2} - \frac{1}{(kr)^3} \right)$$

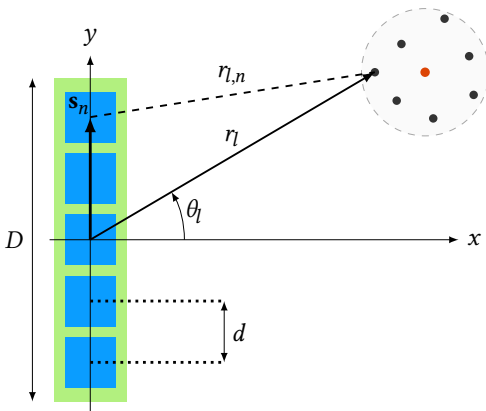
- Near-field region:

$$[\mathbf{a}(r, \theta)]_n = g_n \frac{e^{-jkr_n}}{kr_n}$$

- NLoS propagation:

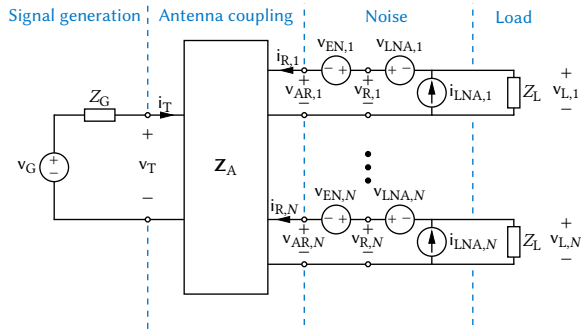
$$\mathbf{h} = \sum_{l=1}^L \alpha_l \mathbf{a}(r_l, \theta_l)$$

$$\mathbf{C}_h = \sum_{l=1}^L \beta_l \mathbf{a}(r_l, \theta_l) \mathbf{a}(r_l, \theta_l)^H$$



Geometry of a ULA deployed along the y axis.

Multiport Communication Theory



Multiport communication model of a SIMO system.

Antenna Coupling

$$\mathbf{Z}_A^{\text{UA}} = \begin{pmatrix} Z_T & \mathbf{0} \\ \mathbf{h} & \mathbf{Z}_R \end{pmatrix} \in \mathbb{C}^{(N+1) \times (N+1)},$$

$$[\mathbf{Z}_R]_{m,n} = Z_{||}(d|m-n|), \quad m \neq n$$

Noise-Free Relation ($\mathbf{v}_L = \mathbf{d}\mathbf{v}_G$)

$$\mathbf{d}^{\text{UA}} = \frac{Z_L}{Z_G + Z_T} \left(Z_L \mathbf{I}_N + \mathbf{Z}_R - \cancel{\frac{\mathbf{h}\mathbf{h}^T}{Z_G + Z_T}} \right)^{-1} \mathbf{h}$$

Unilateral Approximation

$$\frac{\mathbb{E}[\|\mathbf{h}\mathbf{h}^T\|_F]}{|Z_G + Z_T|} \ll \|\mathbf{Z}_L \mathbf{I}_N + \mathbf{Z}_R\|_F \implies \frac{\text{tr}(\mathbf{C}_h)}{|Z_G + Z_T|} \ll \|\mathbf{Z}_L \mathbf{I}_N + \mathbf{Z}_R\|_F$$

Asymptotic Analysis

Intra-array coupling admits the lower bound [2]:

$$\|Z_L \mathbf{I}_N + \mathbf{Z}_R\|_F^2 \geq N|Z_L + R_r|^2 + \underbrace{\frac{9}{4}NR_r^2 \left(\frac{H_{N-1}^{(2)}}{(\kappa d)^2} - \frac{H_{N-1}^{(4)}}{(\kappa d)^4} + \frac{H_{N-1}^{(6)}}{(\kappa d)^6} \right)}_{\text{Generalized Harmonic Numbers} \rightarrow \text{Bounded for fixed } d}$$

Generalized Harmonic Numbers $\rightarrow$ Bounded for fixed d

The reflected component is given by:

$$\frac{\text{tr}(\mathbf{C}_h)}{|Z_G + Z_T|} = \frac{1}{|Z_G + Z_T|} \sum_{l=1}^L \beta_l \|\mathbf{a}(r_l, \theta_l)\|_2^2, \quad \|\mathbf{a}(r_l, \pi/2)\|_2^2 = \frac{R_r^2}{\kappa^2} \left(2 \sum_{n=0}^{\frac{N-1}{2}} \frac{1}{r_l^2 + (nd)^2} - \frac{1}{r_l^2} \right)$$

Discrete Arrays ($D \rightarrow \infty$)

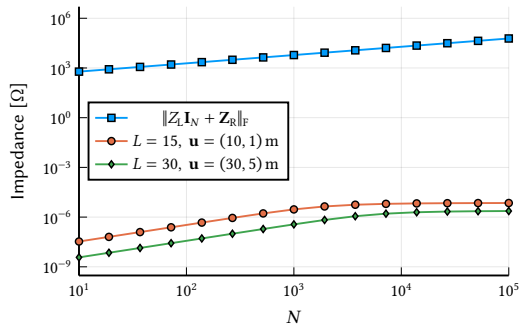
$$\|Z_L \mathbf{I}_N + \mathbf{Z}_R\|_F = O(\sqrt{N}),$$
$$\lim_{N \rightarrow \infty} \|\mathbf{a}(r_l, \pi/2)\|_2^2 = \frac{R_r^2 \pi r_l \coth\left(\frac{\pi r_l}{d}\right)}{d \kappa^2 r_l^2} < +\infty$$

Continuous Arrays ($d = D/N \rightarrow 0$)

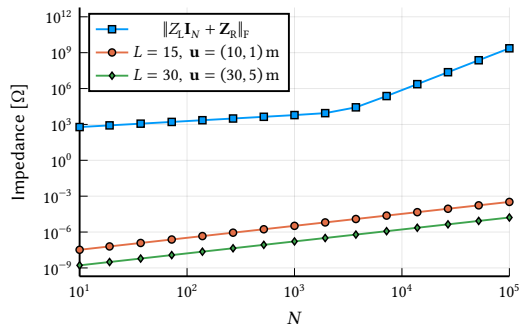
$$\|Z_L \mathbf{I}_N + \mathbf{Z}_R\|_F = O(N^{3.5}),$$
$$\text{tr}(\mathbf{C}_h)/|Z_G + Z_T| = O(N)$$

Numerical Results

Parameter	Value	Parameter	Value
Carrier frequency	$f = 10$ GHz	Scattering sphere radius	$r_s = 3$ m
Amplifier and load impedance	$Z_G = Z_L = 186 - j31.6 \Omega$	Dipole length	$l = \lambda/20$



Intra- and inter-array coupling norms as functions of N for fixed inter-element spacing $d = \lambda/2$.



Intra- and inter-array coupling norms as functions of N for fixed aperture $D = 5$ m.

Additional Scenarios

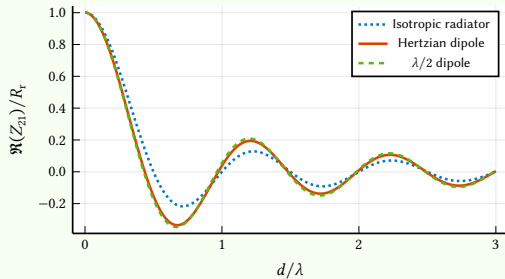
Collinear Dipoles

Toroidal radiation of Hertzian dipoles, with mutual impedance:

$$Z_{||}(r) = 3R_r e^{-j\kappa r} \underbrace{\left(-\frac{1}{(\kappa r)^2} + \frac{j}{(\kappa r)^3} \right)}_{\text{No } 1/r \text{ term}}$$

- Smaller mutual coupling compared to side-by-side dipoles.
- Conclusion: same divergence rate.

Practical Antennas



- Qualitatively similar mutual impedance.
- Inter-array coupling is evaluated in the radiative near field (point sources).

Conclusions

- **Validity in near-field NLoS:** The unilateral approximation is asymptotically satisfied on average in near-field NLoS multipath channels.
- **Fixed d :** Intra-array coupling grows as $O(\sqrt{N})$, while reflected coupling converges to a finite upper bound via Poisson summation as $N \rightarrow \infty$.
- **Fixed D :** Intra-array coupling grows as $O(N^{3.5})$ once $d \leq \lambda/20$, dominating the linear $O(N)$ growth of reflected coupling.
- **LoS consistency:** Divergence rates match prior LoS results and apply to collinear dipoles.

- [1] M. T. Ivrlač and J. A. Nossek, “Toward a circuit theory of communication,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 7, pp. 1663–1683, 2010.
- [2] A. Martí, J. Riba, M. Lamarca, and X. Gràcia, “Asymptotic analysis of near-field coupling in massive MISO and massive SIMO systems,” *IEEE Communications Letters*, vol. 28, no. 8, pp. 1929–1933, 2024.
- [3] A. Martí, A. A. D’Amico, L. Sanguinetti, and X. Gràcia, “The unilateral approximation in near-field communications: From circuit theory to spectral efficiency,” (to be submitted).

This work has been supported by:

- Project MAYTE (PID2022-136512OB-C21) by MICIU/AEI/10.13039/501100011033 and ERDF/EU.
- Grant 2021 SGR 01033 by Departament de Recerca i Universitats de la Generalitat de Catalunya.
- Grant 2023 FI “Joan Oró” 00050 by Departament de Recerca i Universitats de la Generalitat de Catalunya and the ESF+.
- FoReLab Project (Department of Excellence) by the Italian Ministry of Education and Research (MUR).