OFDM Sidelobe Suppression: CSIT-aware Orthogonal Precoding

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Motivation

- **What the Bottleneck is:**
  - Rectangular pulse CP-OFDM has slow sidelobe decay ($\propto f^{-2}$) $\Rightarrow$ high OBR/ACI.
  - Practical designs must deliver *simultaneously*: (i) sharp OBR suppression in a prescribed band $\mathcal{B}$, (ii) strong SER on frequency-selective channels, (iii) low-complexity decoding.
  - Most spectral precoding schemes are CSIT-blind, PSP-OFDM being an exception [1].
- **What we propose:**
  - BSPM-based semi-unitary spectral precoding $\Rightarrow$ OBR mitigation in $\mathcal{B}$.
  - CSIT-driven right-unitary rotation $\Rightarrow$ improved receiver reliability with unchanged PSD/OBR.
  - Receiver-friendly effective channel $\Rightarrow$ SVD/GMD-based rotations to enable ZF/SIC decoding.

1. Signal & Channel Model

- **MC setup:** N-IFFT system with $K \leq N$ active subcarriers; guard interval l_g.
- **Spectral precoding:**
  - At blk. m, map $D \leq K$ syms. $\mathbf{d}_m \in \mathbb{A}^D$ to active tones
$$\mathbf{x}_m = \mathbf{G}\mathbf{d}_m \in \mathbb{C}^K$$
  - $\mathbf{G} \in \mathbb{C}^{K\times D}$: Redun. $R = K - D$ & rate $\lambda = D/K \leq 1$.
- **Closed-form PSD:**
  - Under $\mathsf{E}[\mathbf{d}_m\mathbf{d}_{m-\ell}^{\mathsf{H}}] = \delta[\ell]\mathbf{I}_D$, the PSD expression [2]:
$$P_s(f) \propto \boldsymbol{\phi}^{\mathsf{H}}(f)\,\mathbf{G}\mathbf{G}^{\mathsf{H}}\,\boldsymbol{\phi}(f).$$
  - $\boldsymbol{\phi}^{\mathsf{H}}(f)$: stacked per-subcarrier spectral shapes.
- **Block-fading channel:**
  - Exponential PDP multipath:
$$h_c[n] = e^{-\frac{n}{2\delta l_g}}\,\tilde{h}_c[n],\ 0 \leq n \leq l_g - 1, \tilde{h}_c[n] \sim \mathcal{CN}(0,1)$$
  - The post-FFT channel:
$$\mathbf{H} = \mathrm{diag}\{H_c[k_1], \dots, H_c[k_K]\} \in \mathbb{C}^{K\times K}$$
  - $H_c[k]$: N-DFT of $\{h_c[n]\} \Rightarrow \sum_{k\in\mathcal{K}} |H_c[k]|^2 = K$.

2. Problem Statement & Proposed Design

- **Objective:**
  - Define weighted out-of-band power in $\mathcal{B}$:
$$P_W \triangleq \int_{-\infty}^{+\infty} W(f)\,P_s(f)\,df = \mathrm{Tr}\Big(\mathbf{G}^{\mathsf{H}}\boldsymbol{\Phi}\mathbf{G}\Big),\ \ W(f) \geq 0$$
  - $\boldsymbol{\Phi}$ aggregates the DAC filter and subcarrier responses, & depends on the weighting profile $W(f)$.
  - To avoid the trivial solution $\mathbf{G} = \mathbf{0}$, enforce semi-unitary constraint:
$$\min_{\mathbf{G}} \mathrm{Tr}\Big(\mathbf{G}^{\mathsf{H}}\boldsymbol{\Phi}\mathbf{G}\Big) \ \text{ s.t. } \ \mathbf{G}^{\mathsf{H}}\mathbf{G} = \mathbf{I}_D.$$
- **Solution:**
  - $\mathbf{G}_\star$ spans the eigenspace of the D smallest eigenvalues of $\boldsymbol{\Phi}$,
  - $\mathbf{G}_\star$ & $\mathbf{G}_\star\mathbf{V}^{\mathsf{H}}$ are spectrally equivalent $\Rightarrow$ same PSD/OBR.
- **Received signal:**
  - The received vector (after CP removal and FFT) is
$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{H}\mathbf{G}\mathbf{d} + \mathbf{n}, \qquad \mathsf{E}\Big[\mathbf{n}\mathbf{n}^{\mathsf{H}}\Big] = \sigma^2\mathbf{I}_K$$
  - CSI available at TX (CSIT)
  - Idea: keep $\mathbf{G}_\star$ for spectrum, choose $\mathbf{V}$ (via CSIT) to simplify decoding/improve SER.
- **ZF receiver without CSIT (baseline)**
  - Tx precoder: $\mathbf{G} = \mathbf{G}_\star$
  - RX front-end:
$$\mathbf{G}_\star^{\mathsf{H}}\mathbf{H}^{-1}\mathbf{r} = \mathbf{d} + \mathbf{w} \ \Rightarrow\ \hat{\mathbf{d}} = \mathrm{DEC}_{\mathbb{A}}\Big\{\mathbf{G}_\star^{\mathsf{H}}\mathbf{H}^{-1}\mathbf{r}\Big\}$$
  - Strong noise enhancement on deep fades due to $\mathbf{H}^{-1}$.
- **ZF receiver with CSIT (SVD-based)**
  - Effective channel decomposition:
$$\mathbf{H}\mathbf{G}_\star = \mathbf{U}_c\boldsymbol{\Gamma}_c\mathbf{V}_c^{\mathsf{H}}, \quad \boldsymbol{\Gamma}_c = \mathrm{diag}\{\gamma_1, \dots, \gamma_D\}$$
  - TX precoder:
$$\mathbf{G} = \mathbf{G}_\star\mathbf{V}_c \ \Rightarrow\ \mathbf{r} = \mathbf{U}_c\boldsymbol{\Gamma}_c\mathbf{d} + \mathbf{n}$$
  - RX front-end:
$$\boldsymbol{\Gamma}_c^{-1}\mathbf{U}_c^{\mathsf{H}}\mathbf{r} = \mathbf{d} + \mathbf{w}_c \ \Rightarrow\ \hat{\mathbf{d}} = \mathrm{DEC}_{\mathbb{A}}\Big\{\boldsymbol{\Gamma}_c^{-1}\mathbf{U}_c^{\mathsf{H}}\mathbf{r}\Big\}$$
  - Noise enhancement depends on the γ_i.
- **SIC receiver with CSIT (GMD-based)**
  - Effective channel decomposition
$$\mathbf{H}\mathbf{G}_\star = \gamma\,\mathbf{Q}_c\mathbf{R}_c\mathbf{P}_c^{\mathsf{H}}, \quad \gamma = \sqrt[D]{\gamma_1\gamma_2\cdots\gamma_D}$$
  - TX precoder:
$$\mathbf{G} = \mathbf{G}_\star\mathbf{P}_c \ \Rightarrow\ \mathbf{r} = \gamma\mathbf{Q}_c\mathbf{R}_c\mathbf{d} + \mathbf{n}$$
  - RX front-end:
$$\gamma^{-1}\mathbf{Q}_c^{\mathsf{H}}\mathbf{r} = \mathbf{R}_c\mathbf{d} + \tilde{\mathbf{w}}_c \ \Rightarrow\ \text{SIC decision rule!}$$
  - White post-noise with variance σ^2/γ^2, with possible error propagation at low SNR.

3. System Configuration

- **Simulation setup:**
  - **CP-OFDM:** rectangular pulse; $N = 256$, $l_g = N/8 = 32$, $\Delta_f = \frac{1}{NT_s}$, $K = 129$ subcarriers, symmetric around carrier, 16-QAM modulation.
  - **OBR band:** $\mathcal{B} = \{f : \frac{1}{4T_s} + \frac{\Delta_f}{2} \leq |f| \leq \frac{1}{2T_s}\}$, with $W(f) = \mathbb{1}_{f\in\mathcal{B}}$.
  - **Channel (block-fading):** δl_g = RMS delay spread (samples), $\delta = \{0.01, 0.1\}$
- **Channel response:**

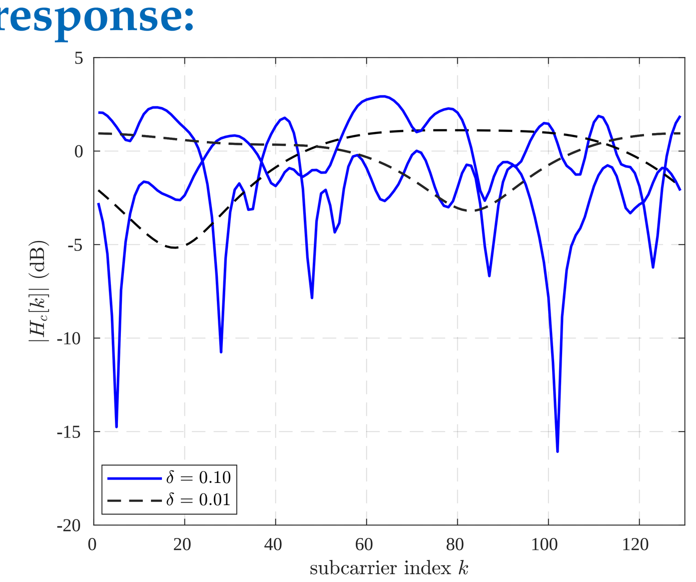

Fig. 1: Channel response $|H_c[k]|$ vs. subcarrier index

4. Performance Evaluation

- **Sidelobe suppression:**

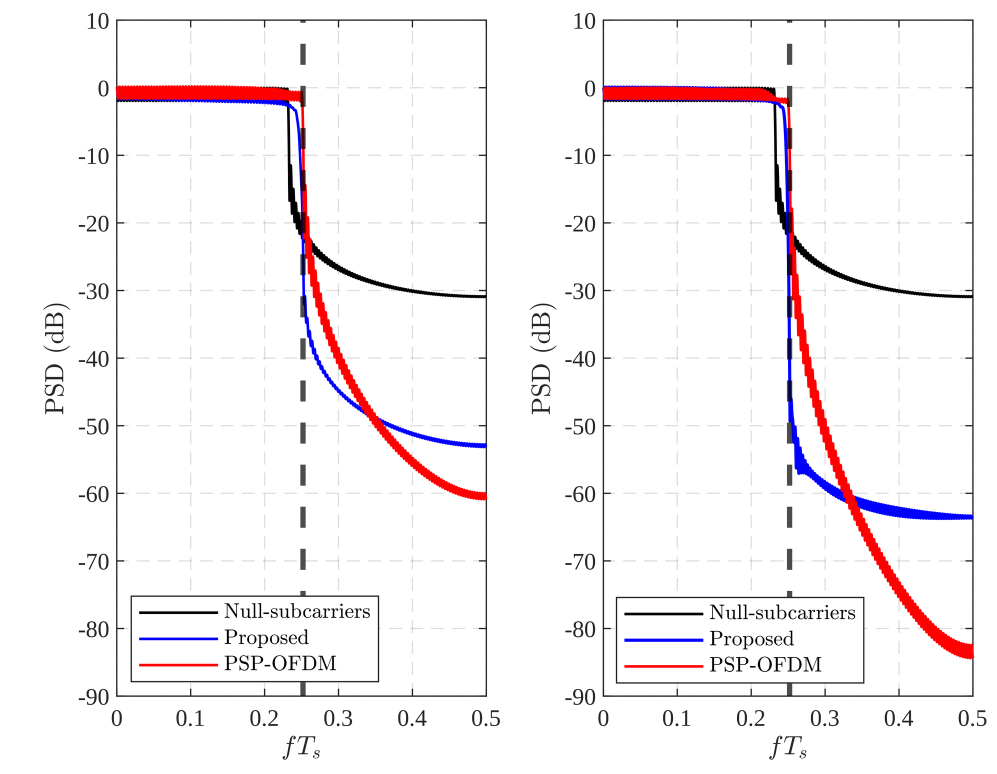

Fig. 2: PSDs of proposed and PSP-OFDM precoders ($\delta = 0.10$); with $R = 6$ (left), and $R = 10$ (right).

- **Error rate:**

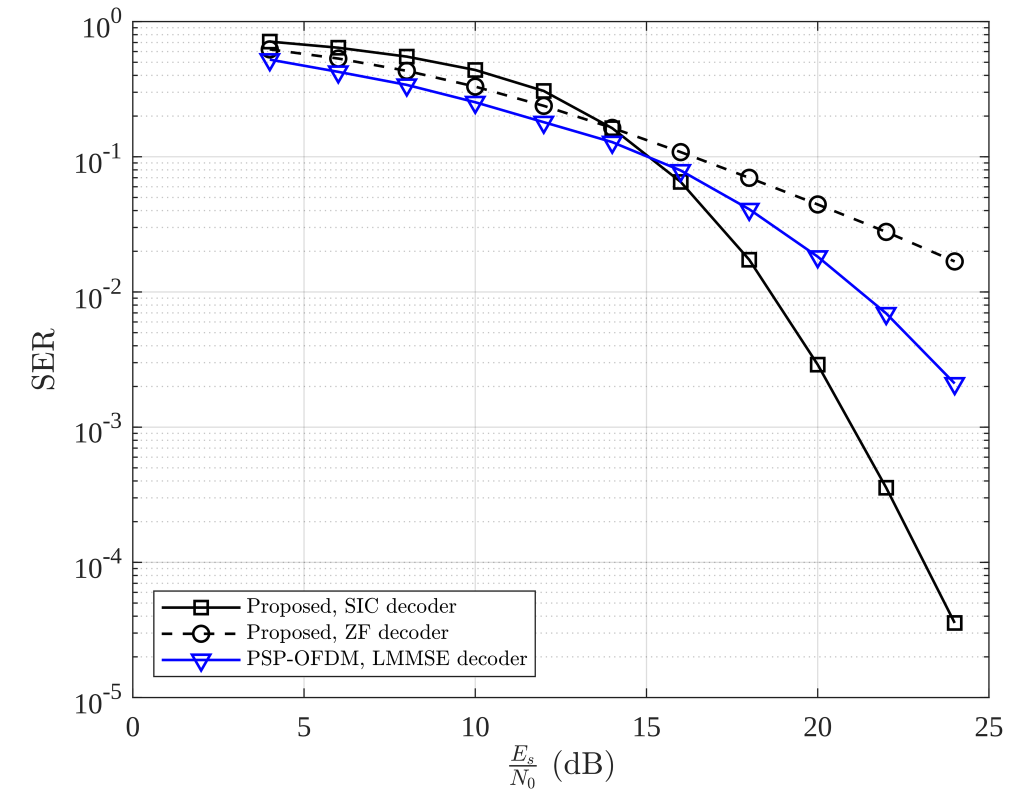

Fig. 3: SER performance of proposed and PSP-OFDM precoders (ZF and SIC), with $\delta = 0.10$ and $R = 6$.

References & Acknowledgment

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[2] R. López-Valcarce, "General form of the power spectral density of multicarrier signals," vol. 26, no. 8, pp. 1755–1759, 2022.

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