

SINGULAR DETECTION IN NONCOHERENT COMMUNICATIONS



Signal Processing
& Communications

MARC VILÀ-INSA¹ and JAUME RIBA²
Signal Processing and Communications Group (SPCOM) – Universitat Politècnica de Catalunya (UPC)

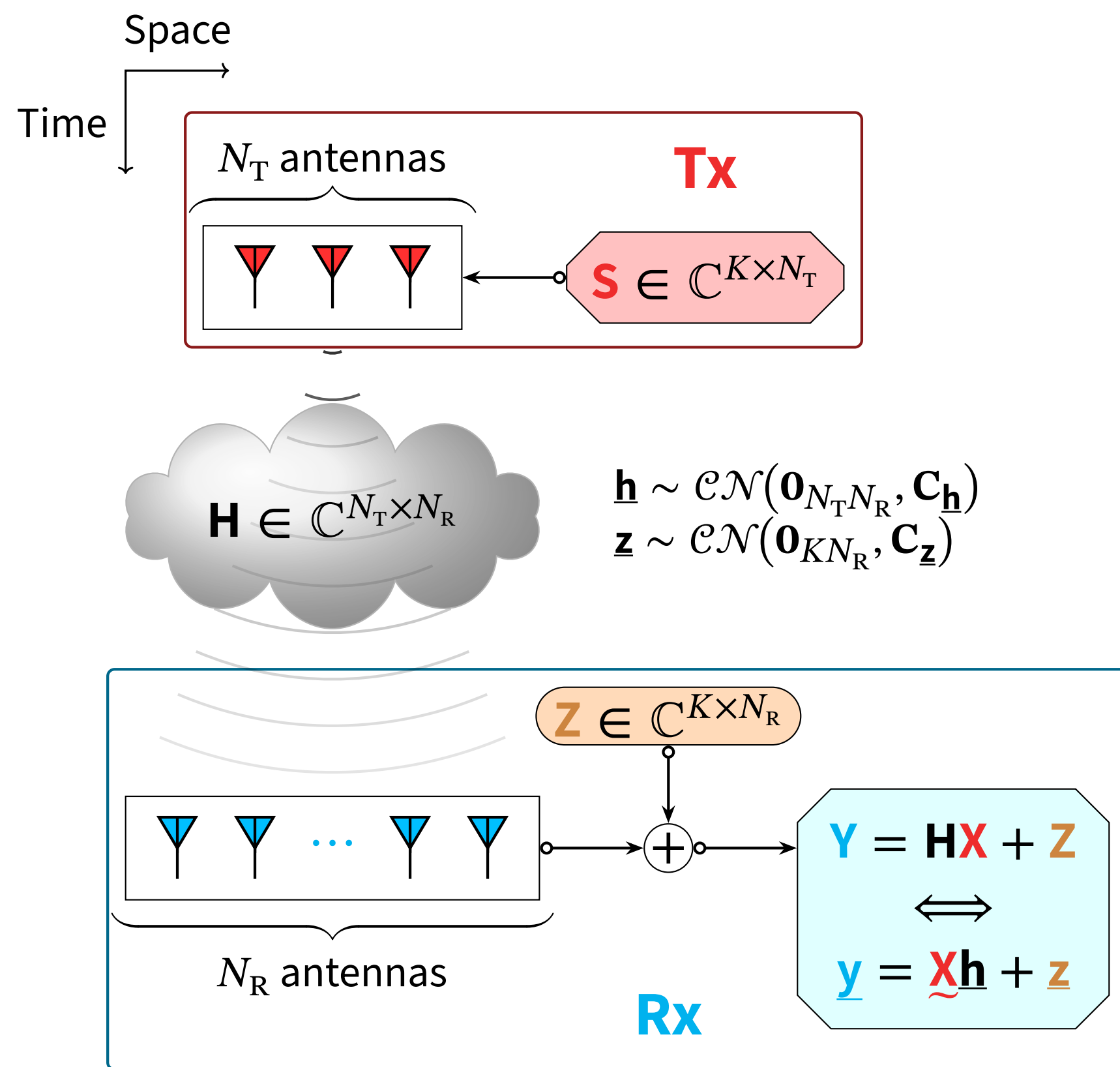


UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
Departament de Teoria del Senyal
i Comunicacions

HIGHLIGHTS

- General analysis of noncoherent communications based on detection theory
- Necessary and sufficient conditions for error-free data transmission:
 - Large array regime
 - High SNR regime
- Extended family of alphabets that achieve *asymptotically singular detection*

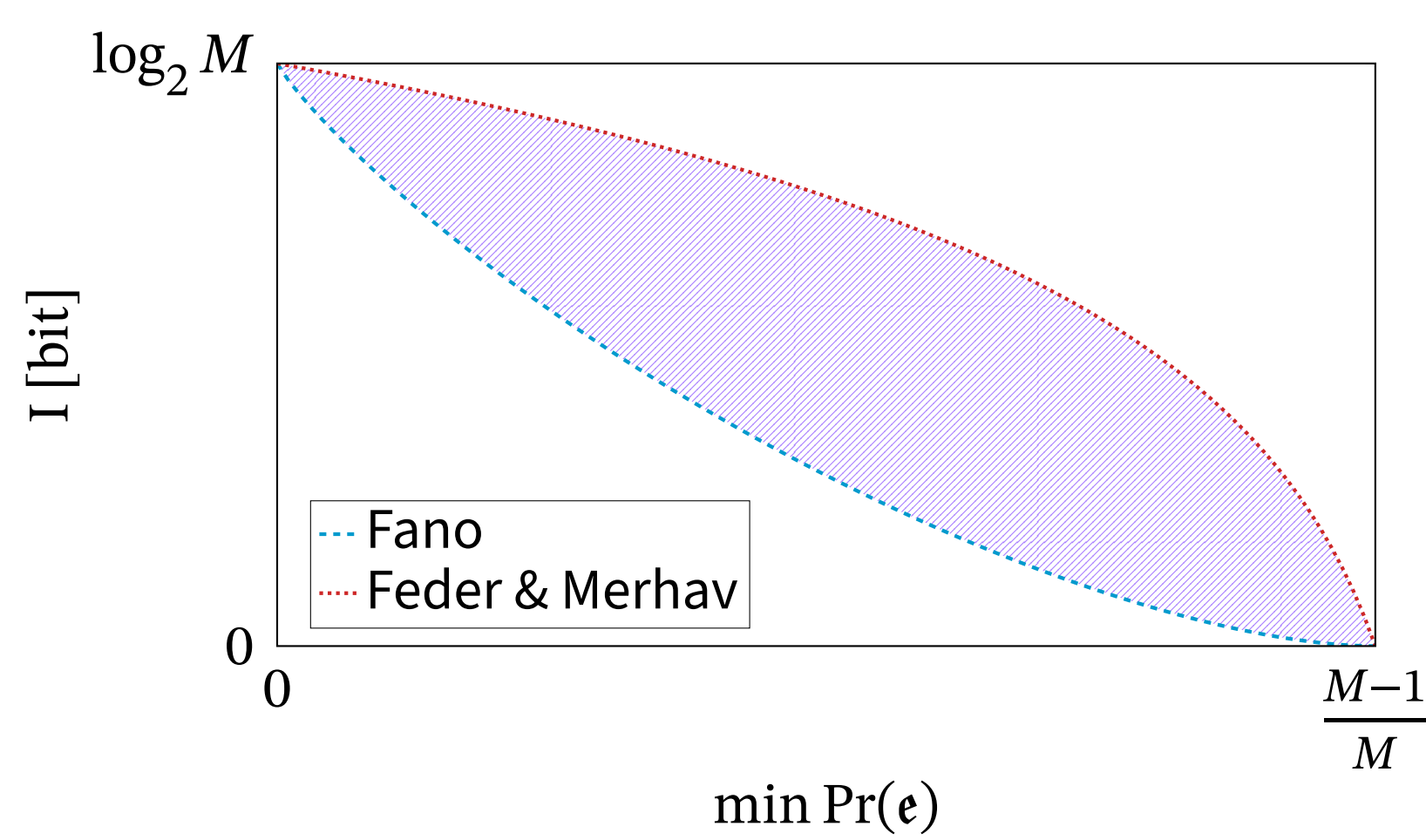
SIGNAL MODEL



- M -ary constellation $\mathcal{S}$, indices $\mathcal{I}$
- Unit power symbols $\mathbf{S}_i \in \mathcal{S}, i \in \mathcal{I}$
- Transmitted signal: $\mathbf{X} \triangleq \sqrt{P_X} \mathbf{S}$
- Power normalizations: $E[\|\mathbf{H}\|_F^2] = N_T N_R, E[\|\mathbf{Z}\|_F^2] = P_Z K N_R$
- SNR at the receiver: $\text{SNR} \triangleq \frac{E[\|\mathbf{H}\mathbf{X}\|_F^2]}{E[\|\mathbf{Z}\|_F^2]}$
- Statistical CSIR
- Correlated Rayleigh fading and Gaussian noise
- Vectorized signal $\mathbf{y} \triangleq \text{vec}(\mathbf{Y}) \in \mathbb{C}^{KN_R}$ and extended codeword $\mathbf{x} \triangleq \mathbf{I}_{N_R} \otimes \mathbf{X} \in \mathbb{C}^{KN_R \times N_T N_R}$

FROM INFORMATION TO DETECTION THEORY

- Detection error probability ($\Pr(\epsilon)$) bounds mutual information between transmitted and received signals ($I(\mathbf{Y}; \mathbf{S})$):



- Pairwise error probability (PEP):

$$\Pr(\mathbf{S}_i \rightarrow \mathbf{S}_j) = \Pr(\ln L_{i,j}(\mathbf{y}) \leq 0 \mid \mathbf{S} = \mathbf{S}_i),$$

where

$$\ln L_{i,j}(\mathbf{y}) \triangleq \ln \frac{f_{\mathbf{y}|\mathbf{S}_i}(\mathbf{y})}{f_{\mathbf{y}|\mathbf{S}_j}(\mathbf{y})}$$

is the log-likelihood ratio between hypotheses i and j .

- Maximum PEP bounds $\Pr(\epsilon)$:

$$\frac{1}{M} \max_{i \neq j \in \mathcal{I}} \Pr(\mathbf{S}_i \rightarrow \mathbf{S}_j) \leq \Pr(\epsilon) \leq (M-1) \max_{i \neq j \in \mathcal{I}} \Pr(\mathbf{S}_i \rightarrow \mathbf{S}_j).$$

- Unconditional approach:

$$(\mathbf{y}|\mathbf{S}_i) \sim \mathcal{CN}(\mathbf{0}_{KN_R}, \mathbf{C}_i \triangleq \mathbf{x}_i \mathbf{C}_h \mathbf{x}_i^H + \mathbf{C}_z).$$

- Maximum likelihood detector:

$$\hat{\mathbf{S}}_{\text{ML}} = \arg \min_{i \in \mathcal{I}} \mathbf{y}^H \mathbf{C}_i^{-1} \mathbf{y} + \ln |\mathbf{C}_i|.$$

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UNIQUE IDENTIFICATION AND SINGULAR DETECTION

Definition 1: Unique identification

An alphabet is *uniquely identifiable* if

$$\mathbf{C}_i \neq \mathbf{C}_j \iff \mathbf{S}_i \neq \mathbf{S}_j \quad \forall (i, j) \in \mathcal{I}.$$

- All noncoherent approaches use uniquely identifiable codes: energy-based, differential, orthogonal, Grassmannian constellations, MOCZ, Borran-like, etc.

Definition 2: Asymptotically singular detection

A noncoherent scheme yields *asymptotically singular detection* (ASD) if it can achieve vanishing error probability as some system parameter grows without bound.

- Useful to characterize the existence of error floors.

LEMMA 1 – NECESSITY OF UNIQUE IDENTIFICATION

Unique identification is a necessary condition for an alphabet to be detected with arbitrarily low error probability.

LARGE ARRAY REGIME

- Related to *channel hardening*.

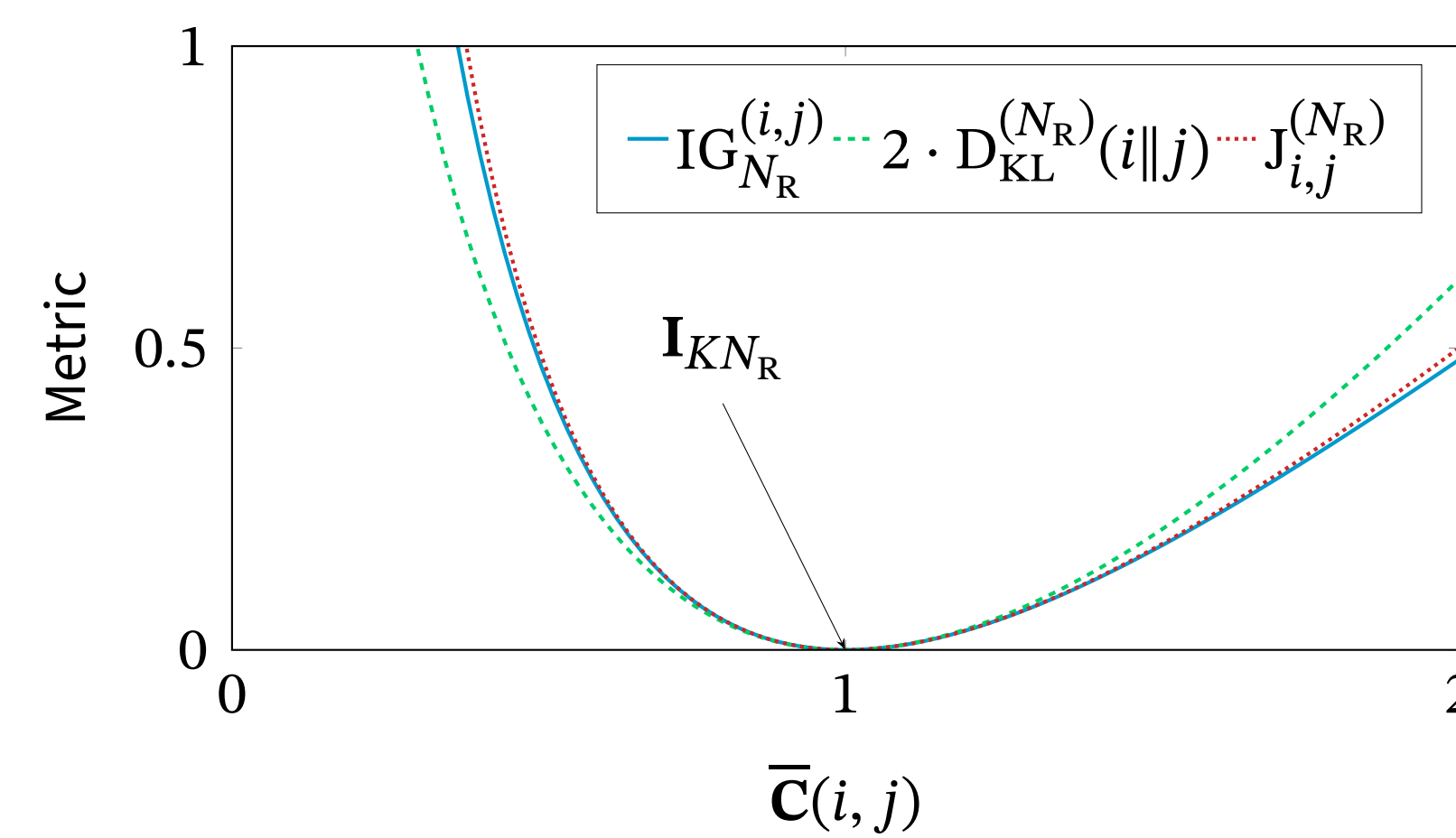
- J-divergence between $f_{\mathbf{y}|\mathbf{S}_i}$ and $f_{\mathbf{y}|\mathbf{S}_j}$:

$$J_{i,j}^{(N_R)} \triangleq D_{\text{KL}}^{(N_R)}(i||j) + D_{\text{KL}}^{(N_R)}(j||i).$$

- It takes the form of a weighted norm:

$$J_{i,j}^{(N_R)} = \text{Tr}[(\mathbf{C}_j^{-1} - \mathbf{C}_i^{-1})(\mathbf{C}_i - \mathbf{C}_j)] = \|\bar{\mathbf{C}}(i, j) - \mathbf{I}_{KN_R}\|_{\bar{\mathbf{C}}(i,j)}^2,$$

where $\bar{\mathbf{C}}(i, j) \triangleq \mathbf{C}_j^{-1/2} \mathbf{C}_i \mathbf{C}_j^{-1/2}$.



PROPOSITION 1 ASD IN THE LARGE ARRAY REGIME

A necessary and sufficient condition for $\mathcal{S}$ to be detected with vanishing error probability as $N_R \rightarrow \infty$ is

$$\lim_{N_R \rightarrow \infty} J_{i,j}^{(N_R)} = \infty \quad \forall \mathbf{S}_i \neq \mathbf{S}_j \in \mathcal{S}. \quad (1)$$

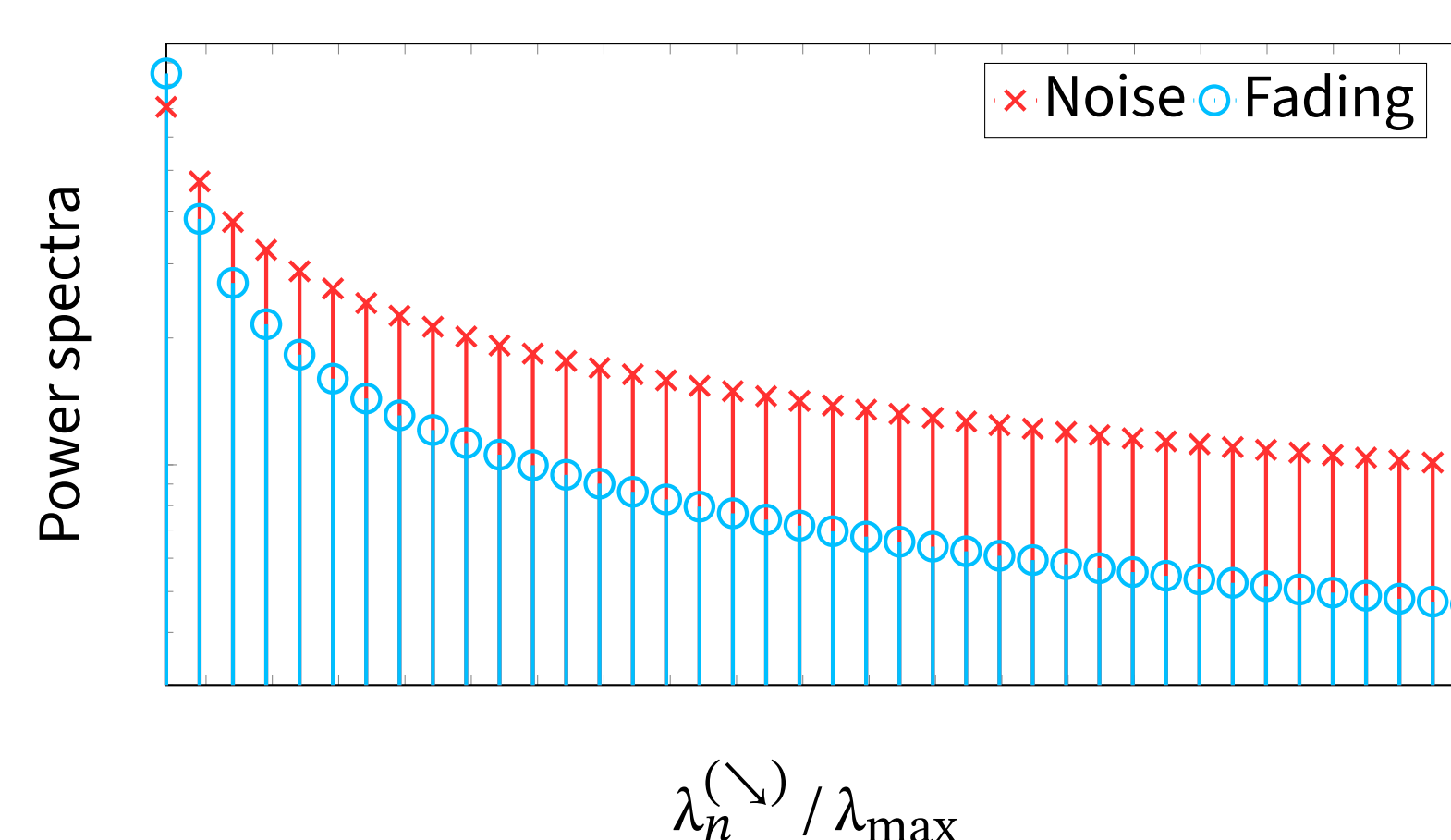
Important considerations

inequality for unique identification	$\iff$	diverging norm for ASD
equality	$\iff$	strong convergence in Hilbert spaces

- ASD does not arise with finite N_R or by increasing received power: it results from deeper interactions between $\mathbf{C}_i$ and $\mathbf{C}_j$.

- Energy-based systems:

$$\begin{cases} \text{UIC} \\ \text{rank}(\mathbf{C}_h) \rightarrow \infty \\ \sum_n \frac{\lambda_n^2(\mathbf{C}_h \mathbf{C}_z^{-1})}{\lambda_{\max}^2(\mathbf{C}_h \mathbf{C}_z^{-1})} \rightarrow \infty \end{cases}$$



- Subspace-based systems: $J_{i,j}^{(N_R)} \propto D_{\text{chordal}}^2(\mathbf{V}_i, \mathbf{V}_j)$,

$$D_{\text{chordal}}^2(\mathbf{V}_i, \mathbf{V}_j) \triangleq |n_i - n_j| + \sum_{n=0}^{n_{\min}-1} \sin^2(\theta_n^{(i,j)}).$$

HIGH SNR REGIME

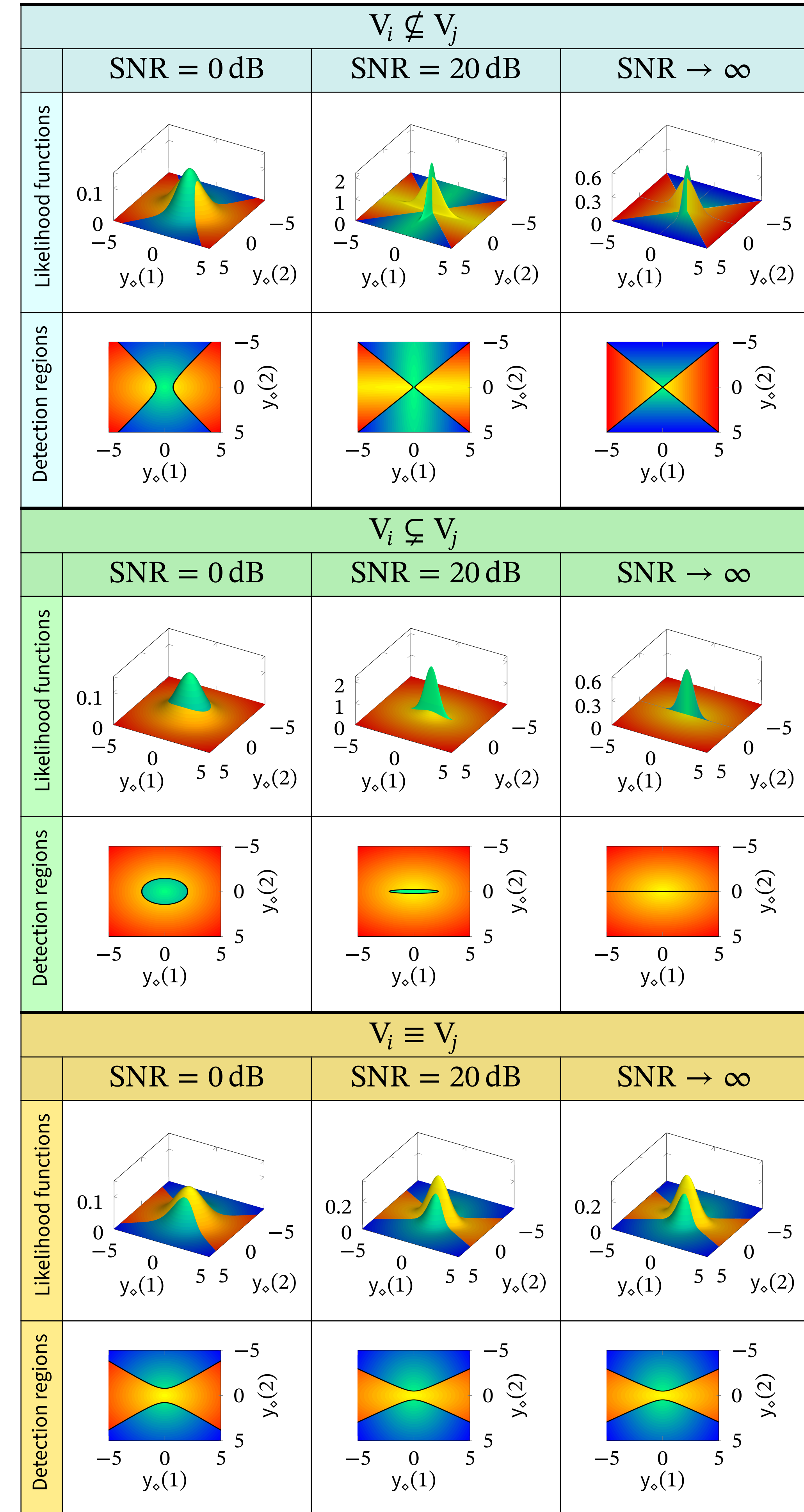
THEOREM 1 – ASD IN THE HIGH SNR REGIME

A necessary and sufficient condition for $\mathcal{S}$ to be detected with vanishing error probability as $\text{SNR} \rightarrow \infty$ is

$$\mathbf{V}_i \neq \mathbf{V}_j \iff \mathbf{S}_i \neq \mathbf{S}_j \quad \forall \mathbf{S}_o \in \mathcal{S}, \quad (2)$$

with $\mathbf{V}_o \triangleq \text{colsp}(\mathbf{S}_o)$.

Intuition:



Implications:

- Spectrum shape of $\mathbf{C}_i$ relative to $\mathbf{C}_j$ becomes irrelevant at high SNR:

$$\begin{matrix} \text{UIC} & & \text{ASD} \\ \mathbf{C}_i \neq \mathbf{C}_j & \xrightarrow{\text{SNR} \rightarrow \infty} & \mathbf{P}_i \neq \mathbf{P}_j \end{matrix}$$

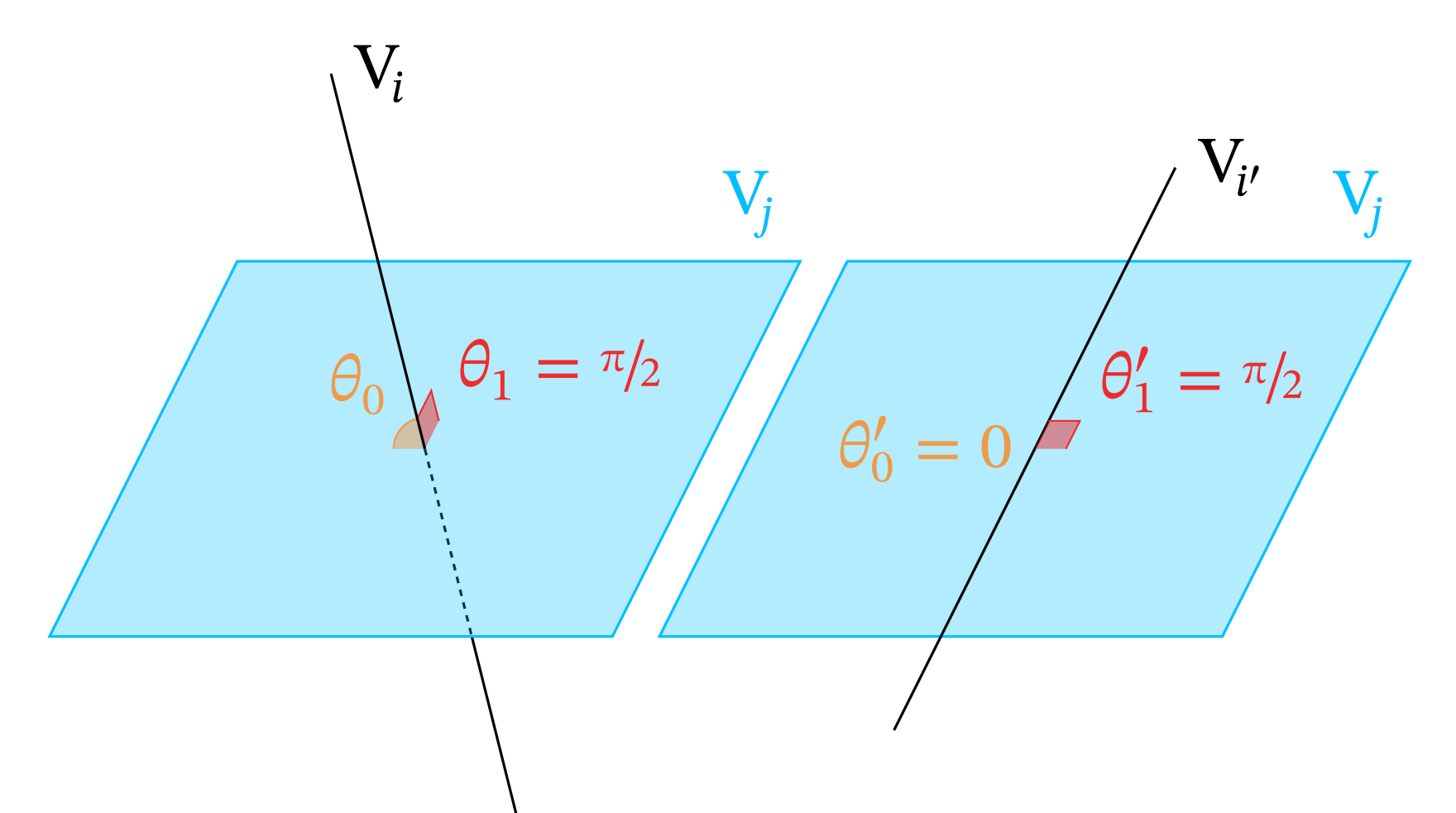
- Schemes that achieve ASD at high SNR: USTM, orthogonal, differential, MOCZ.

- They can all be regarded as Grassmannian constellations.

- Equidimensional codewords are not needed for ASD at high SNR.

- Subspace-based constellations:

$$\mathcal{S}_{\text{subspace}} \triangleq \bigcup_{n=0}^{N_T} \mathcal{S}_n(M_n), \quad \mathcal{S}_n(M_n) \in \mathcal{G}(n, \mathbb{C}^K).$$



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