Multibeam Analog Beamformer Design for Monostatic ISAC under Self-Interference

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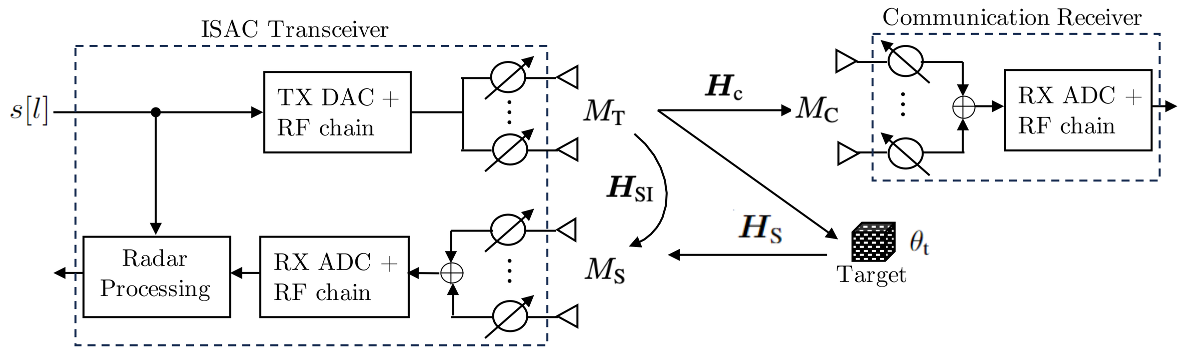
Summary

- This work explores multibeam (dual-functional) fully analog beamforming for joint communication and monostatic sensing under self-interference (SI).
- We provide a semi-analytic optimal solution under total power constraints and its adaptation to constant-modulus (CM) analog beamformers.

Problem Setting



- Assume CM phased arrays: each antenna is connected to a single phase shifter with no individual gain control.
- The ISAC transceiver sends a data stream $\{s[l]\}$ of complex zero-mean, unit-variance symbols. The TX signal reads:

$$\boldsymbol{x}[l] = \boldsymbol{f} s[l] \in \mathbb{C}^{M_\mathsf{T}}$$

- The observation at the co-located radar is given by

$$r[l] = \boldsymbol{w}_\mathsf{s}^H \boldsymbol{H}_\mathsf{S} \boldsymbol{f} s[l] + \underbrace{\boldsymbol{w}_\mathsf{s}^H \boldsymbol{H}_\mathsf{SI} \boldsymbol{f} s[l]}_{\text{Post-combining SI}} + \boldsymbol{w}_\mathsf{s}^H \boldsymbol{n}_\mathsf{s}[l]$$

- Post-combining SI due to weak isolation between TX and sensing arrays and $\ell[\{s[l]\}] >$ target's echo RTT.
- SI degrades radar performance and saturates RF front-end.
- Conventional approach:

$$\min_{\boldsymbol{f},\boldsymbol{w}_\mathsf{s}} \ \mathbb{E}\left\{\boldsymbol{w}_\mathsf{s}^H \boldsymbol{H}_\mathsf{SI} \boldsymbol{f} s[l]\right\}$$

  → Does not protect RF components preceding $\boldsymbol{w}_\mathsf{s}$

- **Solution:** Control *pre-combining* SI

$$P_\mathsf{SI} \triangleq \mathbb{E}\{\|\boldsymbol{H}_\mathsf{SI}\boldsymbol{f} s[l]\|^2\} = \boldsymbol{f}^H \boldsymbol{H}_\mathsf{SI}^H \boldsymbol{H}_\mathsf{SI} \boldsymbol{f}$$

Objective:

$$\begin{aligned}\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \quad & G_\mathsf{tx,comm} \triangleq \boldsymbol{f}^H \boldsymbol{H}_\mathsf{c}^H \boldsymbol{H}_\mathsf{c} \boldsymbol{f} \\ \text{s.t.} \quad & G_\mathsf{tx,sen}(\theta_\mathsf{t}) \triangleq \left|\boldsymbol{f}^H \boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\right|^2 \geq \tau^2 \\ & P_\mathsf{SI} \leq \eta^2 \\ & |f_i| = 1,\ i \in \{1,\dots,M_\mathsf{T}\}\end{aligned}$$

Design under Total Power Constraints

- To gain insight: relax CM constraints

$$|f_i| = 1,\ i \in \{1,\dots,M_\mathsf{T}\} \to \|\boldsymbol{f}\|^2 = M_\mathsf{T}$$

- The design problem becomes

$$\begin{aligned}\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \quad & G_\mathsf{tx,comm} \\ \text{s.t.} \quad & \text{(a) } G_\mathsf{tx,sen}(\theta_\mathsf{t}) \geq \tau^2,\ \text{(b) } P_\mathsf{SI} \leq \eta^2,\ \text{(c) } \|\boldsymbol{f}\|^2 = M_\mathsf{T}\end{aligned}$$

  → QCQP with 3 constraints

- First order optimality condition:

$$\boldsymbol{f}_\star = \sqrt{M_\mathsf{T}}\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \mu_\star \boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \gamma_\star \boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}\right]$$

  → $\mathcal{D}[\boldsymbol{X}]$: dominant unit-norm eigenvector of $\boldsymbol{X}$

  → $\mu_\star$, $\gamma_\star$: optimal Lagrange multipliers for (a) and (b)

- Procedure to attempt to solve the problem:

1. If the solution obtained by relaxing constraints (a) and (b) is feasible:

$$\boldsymbol{f}_\star = \sqrt{M_\mathsf{T}}\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c}\right]$$

   Otherwise, proceed to Step 2.

2. Relaxing either constraint (a) or (b):

   (a) Relax constraint (b) and solve

$$\begin{aligned}\boldsymbol{f}_{(a)} = \ & \arg\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \ G_\mathsf{tx,comm} \\ & \text{s.t. } G_\mathsf{tx,sen} = \tau^2,\ \|\boldsymbol{f}\|^2 = M_\mathsf{T}\end{aligned}$$

   (b) Relax constraint (a) and solve

$$\begin{aligned}\boldsymbol{f}_{(b)} = \ & \arg\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \ G_\mathsf{tx,comm} \\ & \text{s.t. } P_\mathsf{SI} = \eta^2,\ \|\boldsymbol{f}\|^2 = M_\mathsf{T}\end{aligned}$$

   (c) Let $\alpha \triangleq \boldsymbol{f}_{(a)}^H \boldsymbol{H}_\mathsf{SI}^H \boldsymbol{H}_\mathsf{SI} \boldsymbol{f}_{(a)}$ and $\beta \triangleq \boldsymbol{f}_{(b)}^H \boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t})\boldsymbol{f}_{(b)}$. Then,

$$\boldsymbol{f}_\star = \begin{cases}\boldsymbol{f}_{(a)} & \text{if } \alpha \leq \eta^2 \text{ and } \beta \leq \tau^2 \\ \boldsymbol{f}_{(b)} & \text{if } \alpha \geq \eta^2 \text{ and } \beta \geq \tau^2 \\ \arg\max_{\{\boldsymbol{f}_{(a)},\boldsymbol{f}_{(b)}\}} G_\mathsf{tx,comm} & \text{if } \alpha \leq \eta^2 \text{ and } \beta \geq \tau^2\end{cases}$$

   If $\alpha \geq \eta^2$ and $\beta \leq \tau^2$, proceed to Step 3.

3. The optimal $\mu_\star$ and $\gamma_\star$ must simultaneously satisfy $\boldsymbol{f}_\star^H \boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t})\boldsymbol{f}_\star^H = \tau^2$ and $\boldsymbol{f}_\star^H \boldsymbol{H}_\mathsf{SI}^H \boldsymbol{H}_\mathsf{SI} \boldsymbol{f}_\star^H = \eta^2$.

   → Newton-Raphson method

Design under CM constraints

- Restoring the original CM constraints, the problem reads

$$\begin{aligned}\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \quad & G_\mathsf{tx,comm} \\ \text{s.t.} \quad & \text{(a) } G_\mathsf{tx,sen}(\theta_\mathsf{t}) \geq \tau^2,\ \text{(b) } P_\mathsf{SI} \leq \eta^2,\ \text{(c) } |f_i| = 1,\ \forall i\end{aligned}$$

- Some options to enforce CM constraints:

  → Project TPC solution onto the set of CM vectors

  → Replace $|f_i| = 1$ by $|f_i|^2 = 1$ and solve a QCQP with $M_\mathsf{T} + 2$ constraints via SDR

- **Proposal:** Modify the TPC procedure to enforce CM constraints at each iteration:

$$\mathcal{P}_{\mathbb{V}^n}\{\boldsymbol{x}\} = [x_1/|x_1|, \dots, x_n/|x_n|]^T$$

  → At Step 2, find $\boldsymbol{f}_{(a)}$ and $\boldsymbol{f}_{(b)}$ with:

CM Bisection Search$(\boldsymbol{X}, \boldsymbol{Y}, \beta, \epsilon, \omega_{\min}, \omega_{\max})$

- Repeat
  1. $\omega \leftarrow (\omega_{\min} + \omega_{\max})/2$
  2. $\boldsymbol{f} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}\left[(1-\omega)\boldsymbol{X} + \omega\boldsymbol{Y}\right]\}$
  3. if $\boldsymbol{f}^H\boldsymbol{Y}\boldsymbol{f} > \beta$, then $\omega_{\max} \leftarrow \omega$
  4. else $\omega_{\min} \leftarrow \omega$
- Until $|\boldsymbol{f}^H\boldsymbol{Y}\boldsymbol{f} - \beta| \leq \epsilon$

→ At Step 3, find $\boldsymbol{f}_\star$ with:

CM Newton-Raphson$(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}, \beta_1, \beta_2, \epsilon, \boldsymbol{\alpha}_0)$

- $\boldsymbol{\alpha} \leftarrow \boldsymbol{\alpha}_0$
- $\boldsymbol{f} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}[\boldsymbol{X} + \boldsymbol{\alpha}[1]\boldsymbol{Y} - \boldsymbol{\alpha}[2]\boldsymbol{Z}]\}$
- Repeat
  1. $\boldsymbol{u} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}[\boldsymbol{X} + \boldsymbol{\alpha}[1]\boldsymbol{Y} - \boldsymbol{\alpha}[2]\boldsymbol{Z}]\}$
  2. Set

$$\boldsymbol{q}(\boldsymbol{\alpha}) \leftarrow \begin{bmatrix}\boldsymbol{u}^H\boldsymbol{Y}\boldsymbol{u} - \beta_1 \\ \boldsymbol{u}^H\boldsymbol{Z}\boldsymbol{u} - \beta_2\end{bmatrix}.$$

  3. Compute the Jacobian matrix

$$\boldsymbol{J}_{\boldsymbol{q}}(\boldsymbol{\alpha}) \leftarrow \begin{bmatrix}\frac{\partial}{\partial\boldsymbol{\alpha}[1]}q_1(\boldsymbol{\alpha}) & \frac{\partial}{\partial\boldsymbol{\alpha}[2]}q_1(\boldsymbol{\alpha}) \\ \frac{\partial}{\partial\boldsymbol{\alpha}[1]}q_2(\boldsymbol{\alpha}) & \frac{\partial}{\partial\boldsymbol{\alpha}[2]}q_2(\boldsymbol{\alpha})\end{bmatrix}$$

  4. Update $\boldsymbol{\alpha} \leftarrow \boldsymbol{\alpha} - \boldsymbol{J}_{\boldsymbol{q}}^{-1}(\boldsymbol{\alpha})\boldsymbol{q}(\boldsymbol{\alpha})$
  5. $\boldsymbol{f} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}[\boldsymbol{X} + \boldsymbol{\alpha}[1]\boldsymbol{Y} - \boldsymbol{\alpha}[2]\boldsymbol{Z}]\}$
- Until $|\boldsymbol{f}^H\boldsymbol{Y}\boldsymbol{f} - \beta_1| \leq \epsilon$ & $|\boldsymbol{f}^H\boldsymbol{Z}\boldsymbol{f} - \beta_2| \leq \epsilon$

Numerical Example

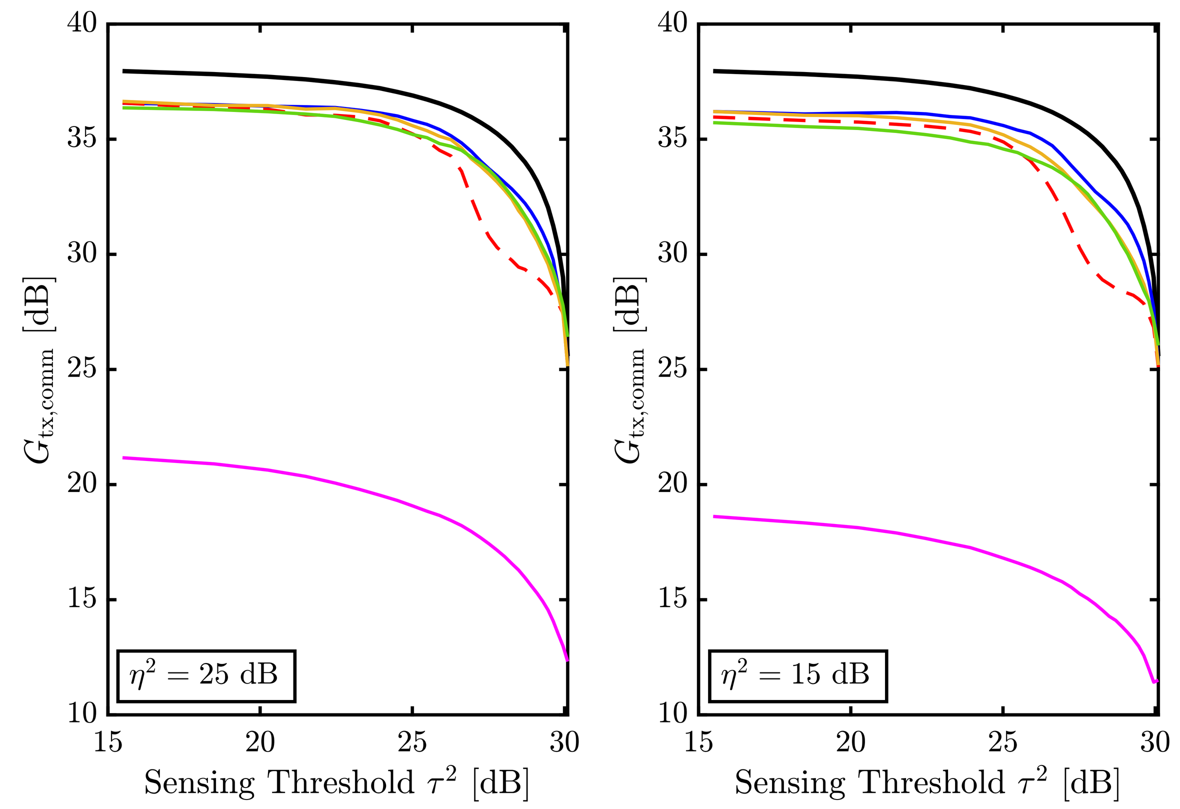
- Proposed with CM (blue)
- Communication-Sensing Upperbound: neglect SI (black)
- Benchmarks:

  → SDR with CM (dashed red)

  → Convex Combination with CM (magenta)

  → Proposed with TPC + Final Projection (orange)

  → SDR with TPC + Final Projection (green)



- SDR with CM

  → Unable to find rank-1 solutions in 40% of the trials

- Proposed with TPC + Final Projection

  → Prob. exceeding SI limit: 20% (left) and 49% (right)

- SDR with TPC + Final Projection

  → Almost surely violates SI constraint

References & Acknowledgements

[1] J. Borras and R. López-Valcarce, "Multibeam analog beamformer design for monostatic isac under self-interference," in *2026 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2026.

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