CONSTELLATION DESIGN FOR QUADRATIC DETECTION IN NONCOHERENT MASSIVE SIMO COMMUNICATIONS

Aniol Martí, Marc Vilà-Insa, Jaume Riba, and Meritxell Lamarca

Signal Processing and Communications Group (SPCOM)
Universitat Politècnica de Catalunya – BarcelonaTech

telecos BCN

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Departament de Teoria del Senyal i Comunicacions

Motivation

A common agreement for next generation communication systems is the deployment of **massive antenna arrays**. Although employing a large number of antennas has many advantages such as reducing fading and increasing diversity, it comes to a cost in terms of CSI acquistion complexity. In order to mitigate the training overhead, **noncoherent communications** have reemerged.

A popular choice among noncoherent detectors are energy detection schemes, specially for low-complexity applications. These systems employ PAM constellations, with the main drawback of exhibiting an error floor at high SNR when the transmitter has no CSI. Instead, if statistical CSI is also assumed at the transmitter, it is possible to design constellations optimized for a specific detector with **no error floor at high SNR**. Precisely, in this work we focus on the design of such constellations under **correlated Rayleigh fading**.

Problem formulation

- Narrowband SIMO system with N antennas at the receiver.
- Statistical CSIR and CSIT.
- M-ary PAM constellation $\mathcal{X} = \{\sqrt{\varepsilon_1} \triangleq x_1, \dots, \sqrt{\varepsilon_M} \triangleq x_M\}$.

Received signal after preprocessing:

$$\mathbf{r} = \mathbf{h}x + \mathbf{z}, \quad \mathbf{h} \sim \mathcal{CN}(\mathbf{0}_N, \boldsymbol{\Gamma}),\ \mathbf{z} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{I}_N). \tag{1}$$

Under white noise and isotropic channel, ML detection can be decomposed as:

1. Computation of a quadratic statistic of data: $\hat{\varepsilon}(\mathbf{r})$.
2. One-dimensional decision problem: $\hat{x}(\hat{\varepsilon})$.

$$\hat{\varepsilon}(\mathbf{r}) = \mathbf{r}^{\mathrm{H}}\mathbf{A}\mathbf{r} + c$$

Different detection schemes can be obtained by properly designing $\mathbf{A}$ [1]:

- Energy detector (ED): $\mathbf{A}_{\mathrm{ED}} \triangleq \mathbf{I}_N / \mathrm{tr}(\boldsymbol{\Gamma})$.
- High SNR (HSNR): $\mathbf{A}_{\mathrm{HSNR}} \triangleq \boldsymbol{\Gamma}^{-1}/N$.
- Best quadratic unbiased estimator (BQUE): $\mathbf{A}_{\mathrm{BQUE}} \triangleq \boldsymbol{\Gamma}\mathbf{C}_{\mathbf{r}|\varepsilon}^{-2} \big/ \left\|\boldsymbol{\Gamma}\mathbf{C}_{\mathbf{r}|\varepsilon}^{-1}\right\|_{\mathrm{F}}^2$.

Leveraging the CLT, when $N \to \infty$:

$$\hat{\varepsilon}(\mathbf{r}|\varepsilon) \xrightarrow{\mathrm{d}} \mathcal{N}(\varepsilon,\ \|\mathbf{A}(\varepsilon\boldsymbol{\Gamma} + \mathbf{I}_N)\|_{\mathrm{F}}^2). \tag{2}$$

Our goal is to design optimized constellations for such receivers.

Constellation design

From (2), an approximation for the SER can be obtained:

$$\mathrm{P}_\epsilon(\mathbf{A}, \boldsymbol{\varepsilon}, \boldsymbol{\tau}) \approx \frac{1}{M}\left(\sum_{i=2}^{M} \mathrm{Q}\left(\frac{\varepsilon_i - \tau_{i-1}}{\|\mathbf{A}(\varepsilon_i\boldsymbol{\Gamma} + \mathbf{I}_N)\|_{\mathrm{F}}}\right) + \sum_{j=1}^{M-1} \mathrm{Q}\left(\frac{\tau_j - \varepsilon_j}{\|\mathbf{A}(\varepsilon_j\boldsymbol{\Gamma} + \mathbf{I}_N)\|_{\mathrm{F}}}\right)\right)$$

where τ_i denotes the threshold between symbols ε_i and ε_{i+1}.

The constellation design process consists in minimizing the error probability subject to the constraints imposed by the power-limited PAM structure. Formally,

$$\begin{aligned} \min_{\boldsymbol{\varepsilon}, \boldsymbol{\tau}} \quad & \mathrm{P}_\epsilon(\mathbf{A}, \boldsymbol{\varepsilon}, \boldsymbol{\tau}) \\ \text{s.t.} \quad & \boldsymbol{\varepsilon}, \boldsymbol{\tau} \geq \mathbf{0}, \\ & \varepsilon_1 = 0, \\ & \textstyle\sum_{i=1}^{M} \varepsilon_i = M, \\ & \varepsilon_i \leq \tau_i,\ i = 1, \dots, M-1, \\ & \tau_i \leq \varepsilon_{i+1},\ i = 1, \dots, M-1. \end{aligned} \tag{3}$$

Simulation results have been obtained for an exponential correlation model:

$$[\mathbf{C}_{\mathbf{h}}]_{k,l} = \rho^{|k-l|}, \quad \rho \in [0, 1),\ 1 \leq k, l \leq N. \tag{4}$$

In Figure 2 and Figure 3 it can be observed that **the proposed constellation design improves or matches the performance of the KLD optimization method** proposed in [2] for all detectors and correlation levels.

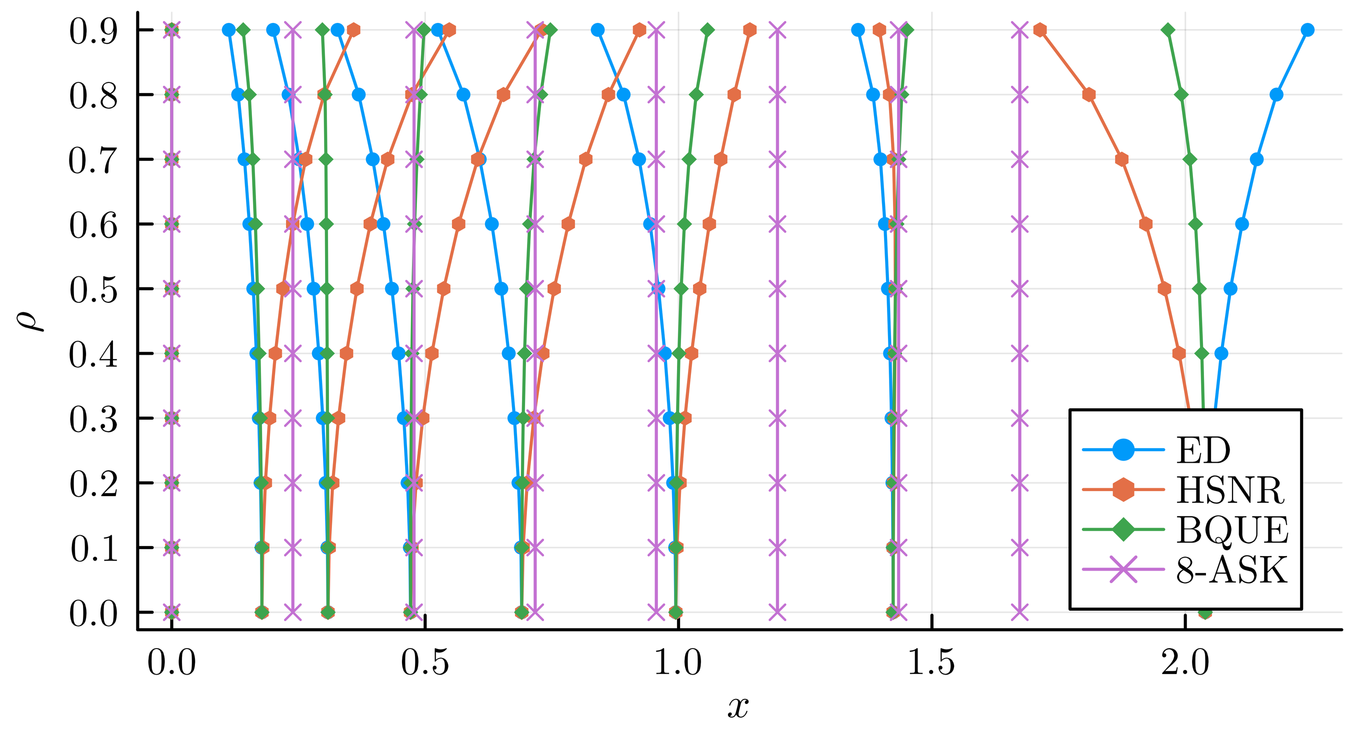

Figure 1. Comparison of 8-ASK and ED, HSNR and BQUE optimized constellations at 15 dB and $N = 256$ for different levels of channel correlation. Given that the variance of each $\hat{\varepsilon}(\mathbf{r}|\varepsilon)$ increases with the symbol amplitude, low-power symbols are closer than those of the 8-ASK, whereas high-power ones are further apart.

Table 1. Summary of the simulation setup and results.

Detector	Solid line	Dashed line	Best constellation
ED	ED-SER	KLD	ED-SER
HSNR	HSNR-SER	KLD	HSNR-SER
BQUE	BQUE-SER	KLD	BQUE-SER/KLD
ML	BQUE-SER	KLD	BQUE-SER

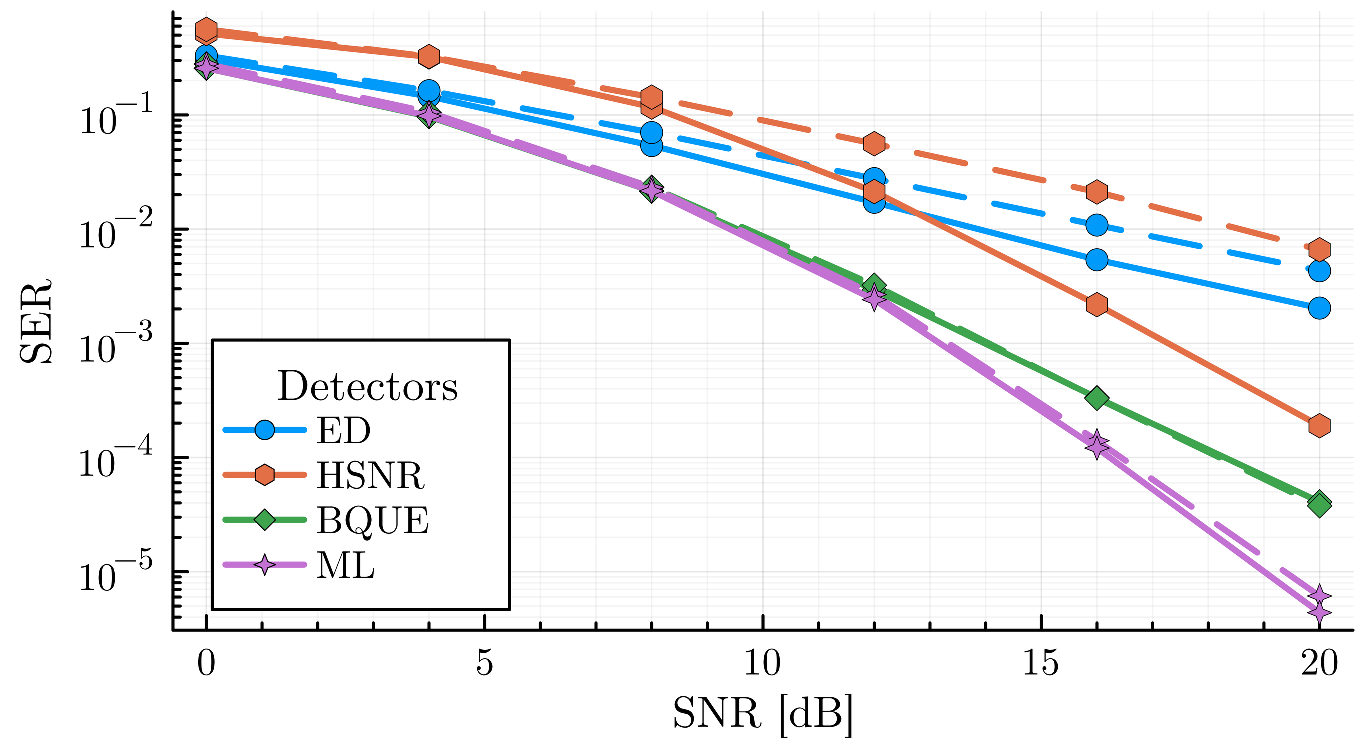

Figure 2. Error probability at different SNR levels with $N = 128$ and $\rho = 0.7$. Note the lack of error floor at high SNR.

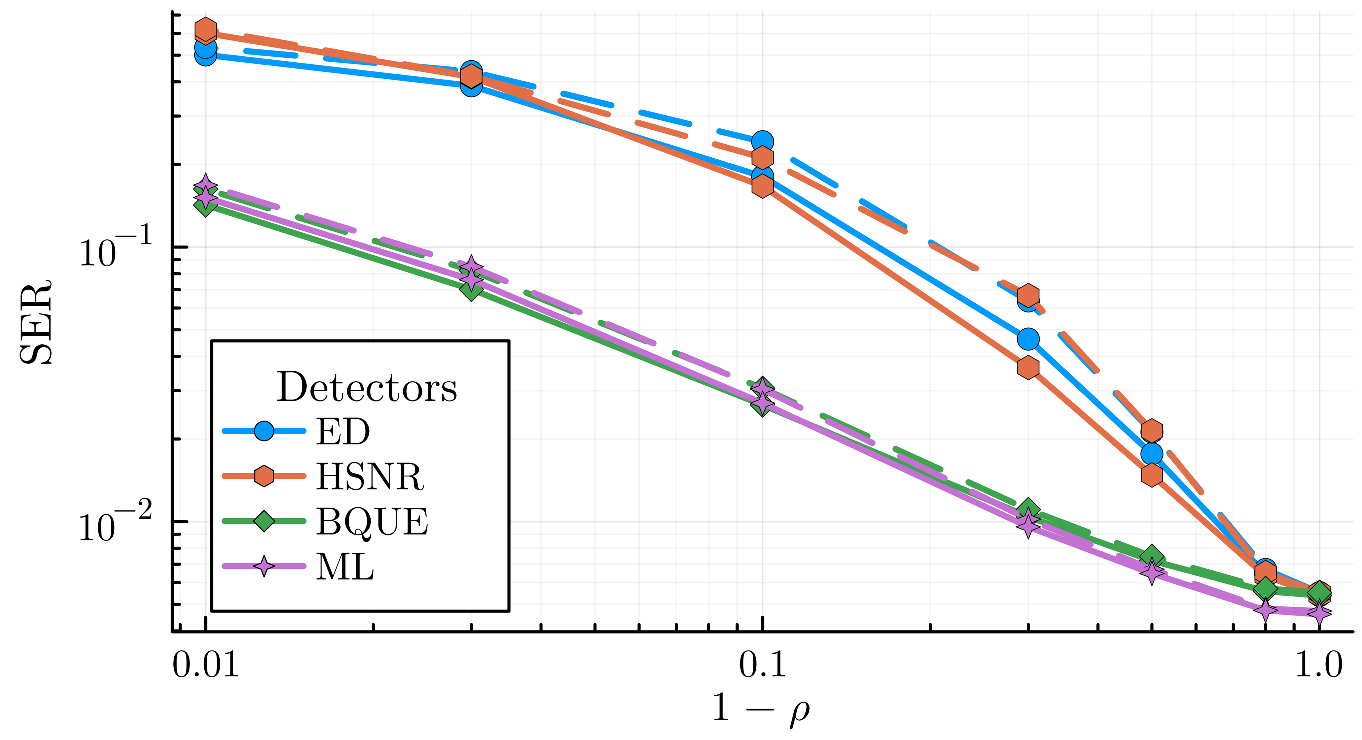

Figure 3. Error probability for different channel correlations with $N = 64$ at a SNR of 15 dB.

References

[1] M. Vilà-Insa, A. Martí, J. Riba, and M. Lamarca, "Quadratic detection in noncoherent massive SIMO systems over correlated channels," *IEEE Trans. Wireless Commun.*, 2024.

[2] W. Han, Z. Dong, H. Chen, and X. Gao, "Constellation design for energy-based noncoherent massive SIMO systems over correlated channels," *IEEE Wireless Commun. Lett.*, vol. 11, no. 10, pp. 2165–2169, Oct. 2022.

Acknowledgments

This work was (partially) funded by project MAYTE (PID2022-136512OB-C21) by MICIU/AEI/10.13039/501100011033 and ERDF/EU grant 2021 SGR 01033 and grant 2022 FI SDUR 00164 by Departament de Recerca i Universitats de la Generalitat de Catalunya and grant 2023 FI "Joan Oró" 00050 by Departament de Recerca i Universitats de la Generalitat de Catalunya and the ESF+.

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN