A CYCLOSTATIONARY PERSPECTIVE ON NONCOHERENT SIMO COMMUNICATIONS

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Highlights

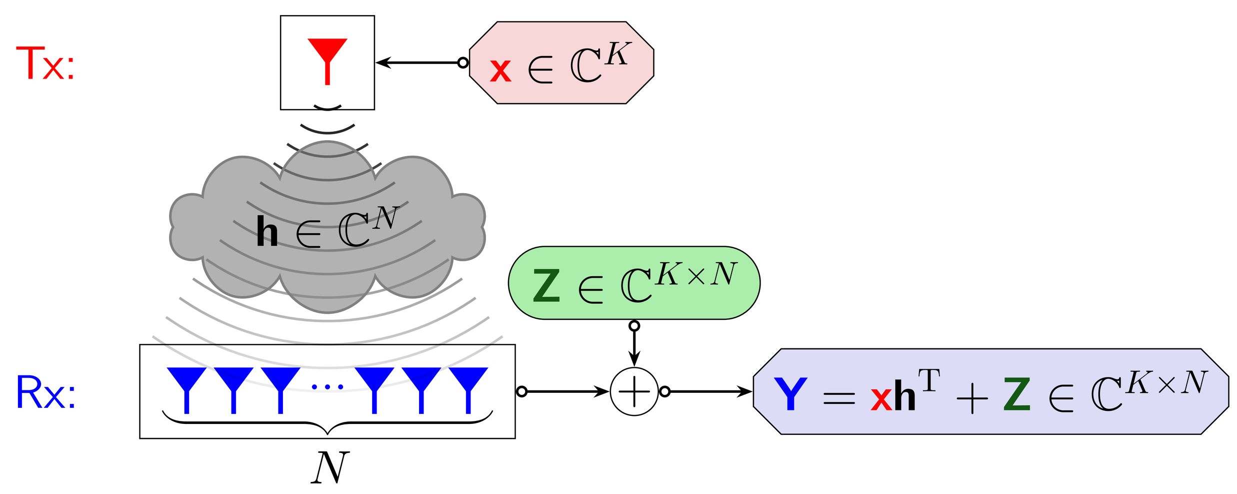
- **Objective:** To characterize statistical properties of the received signal in a noncoherent massive SIMO setting.
- **Approach:**
  - Vectorizing the time-space signal reveals a cyclostationary (CS) process.
  - Its asymptotic structure can be effectively exploited in the Karhunen-Loève (KL) domain.
- **Outcome:** This spectral representation provides insights on noncoherent detection with large arrays and unveils the fundamental parameters that play a role in it, while pointing at simplified implementations.

Background

- It is agreed among the scientific community that future wireless communications will be built upon massive MIMO technologies.
- In many scenarios, the acquisition of reliable instantaneous channel state information (CSI) becomes cumbersome and can degrade spectral efficiency of coherent systems.
- On the contrary, noncoherent communications do not require instantaneous CSI, instead relying on statistical knowledge of the channel.
- To effectively exploit this *statistical CSI*, the receiver can leverage various properties of fading, such as its spatial stationarity.
- Through the use of massive arrays, we access the asymptotic spectral properties of noncoherent SIMO systems, mirroring well-known ideas from time-series analysis in the spatial domain.
- We develop a technique to uncorrelate received data and distill the signal properties that are essential to the detection problem.

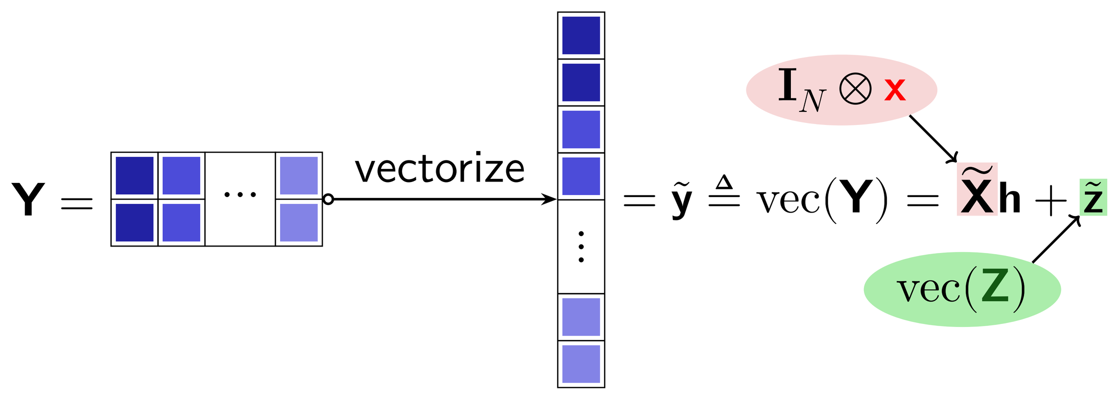
Problem statement

- Point-to-point SIMO, Rx has N antennas and statistical CSI.
- Correlated Rayleigh channel ($\mathbf{h} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{C}_\mathbf{h})$), block flat fading, coherence time K.
- Tx sends an equiprobable codeword $\mathbf{x}_i \in \mathbb{C}^K$ from a finite alphabet $\mathcal{X}$ during a coherence block. For convenience, we parameterize it as $\mathbf{x}_i \triangleq \sqrt{K\gamma_i}\mathbf{u}_i$.
- Additive noise $\mathbf{Z}$ with i.i.d. components $[\mathbf{Z}]_{r,c} \sim \mathcal{CN}(0, \mathrm{P}_\mathbf{Z})$.
- Received signal in complex baseband time-space notation:



Maximum likelihood (ML) detection:

- To derive the (unconditional) likelihood function of the received signal, we must *vectorize* $\mathbf{Y}$ columnwise:



- The resulting received signal is $\tilde{\mathbf{y}}|\mathbf{x}_i \sim \mathcal{CN}(\mathbf{0}_{KN}, \mathbf{C}_i \triangleq \mathbf{C}_\mathbf{h} \otimes \mathbf{x}_i\mathbf{x}_i^\mathrm{H} + \mathrm{P}_\mathbf{Z}\mathbf{I}_{KN})$.
- ML error probability is commonly bounded by the simpler pairwise error probability between codewords $\mathbf{x}_a$ and $\mathbf{x}_b$: $\mathrm{P}_{a\to b} \triangleq \Pr\{\mathrm{L}_{a,b}(\tilde{\mathbf{y}}) \leq 0 | \mathbf{x} = \mathbf{x}_a\}$. The term $\mathrm{L}_{a,b}$ is the log-likelihood ratio (LLR) between hypotheses:

$$\mathrm{L}_{a,b}(\tilde{\mathbf{y}}) \triangleq \ln \frac{\mathrm{f}_{\tilde{\mathbf{y}}|\mathbf{x}_a}(\tilde{\mathbf{y}})}{\mathrm{f}_{\tilde{\mathbf{y}}|\mathbf{x}_b}(\tilde{\mathbf{y}})} = \tilde{\mathbf{y}}^\mathrm{H}(\mathbf{C}_b^{-1} - \mathbf{C}_a^{-1})\tilde{\mathbf{y}} + \ln \frac{|\mathbf{C}_b|}{|\mathbf{C}_a|}. \quad (1)$$

- The most relevant signal characteristics in the detection problem are not directly recognizable from the time-space formulation (1).

> The fundamental structure of the problem will emerge if we represent (1) in the KL domain asymptotically ($N \to \infty$).

Statistical properties of $\tilde{\mathbf{y}}$:

- Assuming channel fading is *spatially stationary* (Figure 1), the resulting process $\mathrm{y}_i(n) \triangleq [\tilde{\mathbf{y}}|\mathbf{x}_i]_n$ will display a CS behavior of period K (Figure 2).

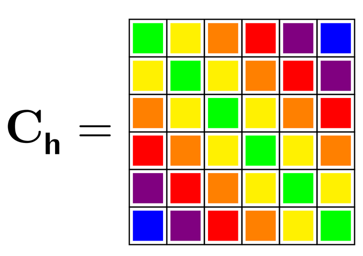


Figure 1. Toeplitz covariance matrix corresponding to a stationary process.

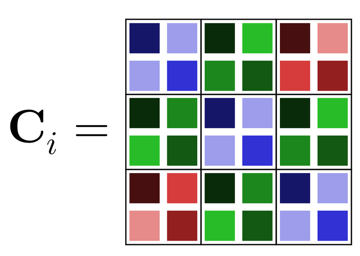


Figure 2. K-Toeplitz ($K = 2$) covariance matrix corresponding to a CS process.

Spectral analysis of the received signal

- A CS process $\mathrm{y}_i(n)$ of period K can be expanded onto its KL basis $\{\phi_i^{(k)}(n,\sigma)\}$ as:

$$\mathrm{y}_i(n) \triangleq \sum_{k=0}^{K-1} \int_0^{\frac{1}{K}} \phi_i^{(k)}(n,\sigma)\mathrm{d}\mathrm{y}_i^{(k)}(\sigma), \quad (2)$$

where $\mathrm{d}\mathrm{y}_i^{(k)}(\sigma)$ are uncorrelated increments in the KL spectral domain.
- In [1], [2] the asymptotic KL basis of $\mathrm{y}_i(n)$ was derived in terms of its *cyclic spectrum matrix (CSM)*, $\mathbf{S}_i(\sigma) \in \mathbb{C}^{K\times K}$. It can be constructed from its spectral correlation in the Cramér-Loève (CL) domain, as shown in Figure 3.

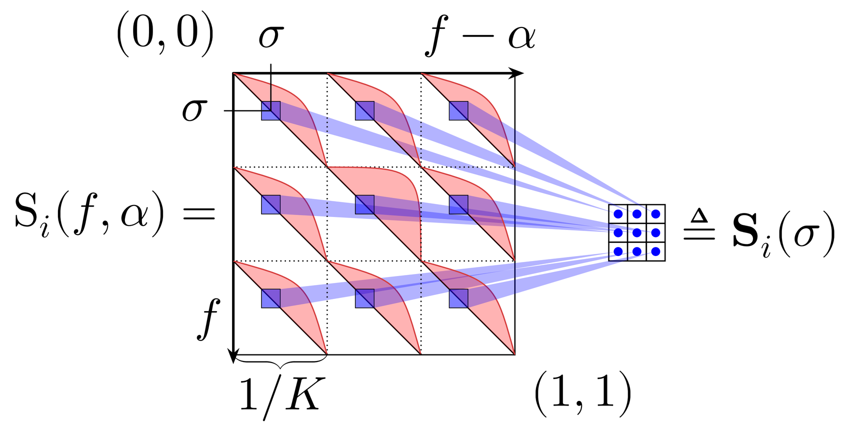


Figure 3. Graphical representation of the construction of $[\mathbf{S}_i(\sigma)]_{r,c}$ from the spectral correlation of $\mathrm{y}_i(n)$ at frequencies $(\sigma + \frac{r}{K}, \sigma + \frac{c}{K})$, *i.e.* $\mathrm{S}_i(\sigma + \frac{r}{K}, \frac{r-c}{K})$. It is only defined across δ-ridges spaced by $1/K$ both horizontally and vertically.

- Using these ideas, we can express the LLR (1) as $N \to \infty$ in the KL domain:

$$\mathrm{L}_{a,b}(\mathrm{d}\mathbf{y}_i) = \int_0^{\frac{1}{K}} \left(\mathrm{d}\mathbf{y}_i^\mathrm{H}\mathbf{\Psi}_i^\mathrm{H}(\mathbf{D}_b^{-1} - \mathbf{D}_a^{-1})\mathbf{\Psi}_i\mathrm{d}\mathbf{y}_i\right)(\sigma) + \ln\frac{|\mathbf{D}_b(\sigma)|}{|\mathbf{D}_a(\sigma)|}\mathrm{d}\sigma \quad (3)$$

where $\mathrm{d}\mathbf{y}_i(\sigma) \triangleq [\mathrm{d}\mathrm{y}_i^{(0)}(\sigma), \mathrm{d}\mathrm{y}_i^{(1)}(\sigma), \dots, \mathrm{d}\mathrm{y}_i^{(K-1)}(\sigma)]$ and $\mathbf{\Psi}_i \triangleq [\mathbf{u}_i\ \mathbf{U}_i^\perp] \in \mathbb{C}^{K\times K}$.
- This representation displays the fundamental structures involved in the detection problem; mainly

$$\mathbf{D}_\diamond(\sigma) \triangleq \gamma_\diamond \mathrm{S}_\mathbf{h}(K\sigma)K\mathbf{P}_\diamond + \mathrm{P}_\mathbf{Z}\mathbf{I}_K \quad , \quad \diamond = a,b. \quad (4)$$

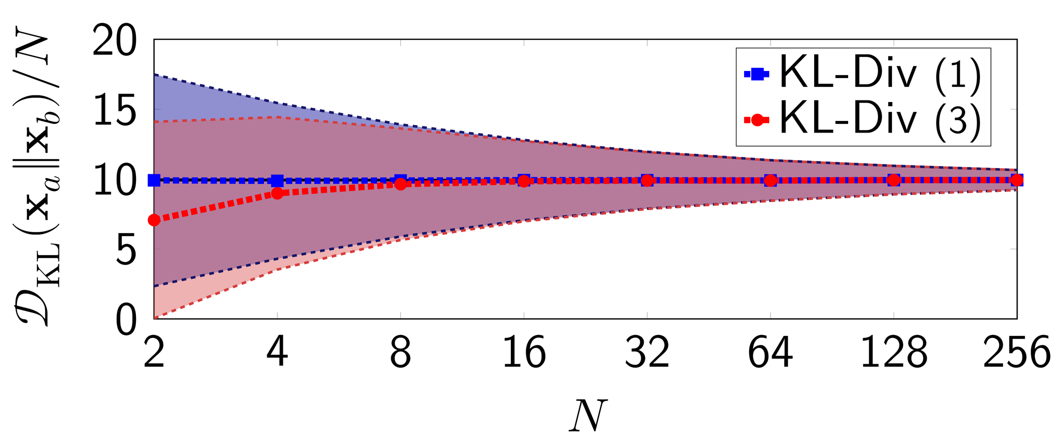


Figure 4. Normalized KL-Div estimated from (1) and (3) by averaging 10^4 Monte Carlo rounds.

Likelihood ratio structure analysis

- Using the relationship between the CL and KL representations of $\mathrm{y}_i(n)$ [2], expression (3) can be decomposed into four clear constituents:

$$\mathrm{L}_{a,b}(\mathrm{d}\mathbf{y}_i) = \ell_{a,b}(\mathbf{x}_i) + \ell_{a,b}(\mathbf{z}) + \ell_{a,b}(\mathbf{x}_i, \mathbf{z}) + \kappa_{a,b} \quad (5)$$

- **Signal term:** $\mathrm{E}[\ell_{a,b}(\mathbf{x}_i)] = \int_0^{\frac{1}{K}} \mathbf{u}_i^\mathrm{H}(|\rho_a(\sigma)|^2\mathbf{P}_a - |\rho_b(\sigma)|^2\mathbf{P}_b)\mathbf{u}_i\Gamma_i(\sigma)\mathrm{d}\sigma$
- **Noise term:** $\mathrm{E}[\ell_{a,b}(\mathbf{z})] = \int_0^{\frac{1}{K}} (|\rho_a(\sigma)|^2 - |\rho_b(\sigma)|^2)\mathrm{d}\sigma$
- **Signal-noise term:** $\mathrm{E}[\ell_{a,b}(\mathbf{x}_i, \mathbf{z})] = 0$
- **Constant term:** $\kappa_{a,b} = \int_0^{\frac{1}{K}} \ln\frac{\Gamma_b(\sigma)+1}{\Gamma_a(\sigma)+1}\mathrm{d}\sigma$

- Terms $\Gamma_\diamond(\sigma)$ and $|\rho_\diamond(\sigma)|^2$ are, respectively, the SNR and *squared spectral coherence* of process $\mathrm{y}_\diamond(n)$ at point σ in the KL spectrum.

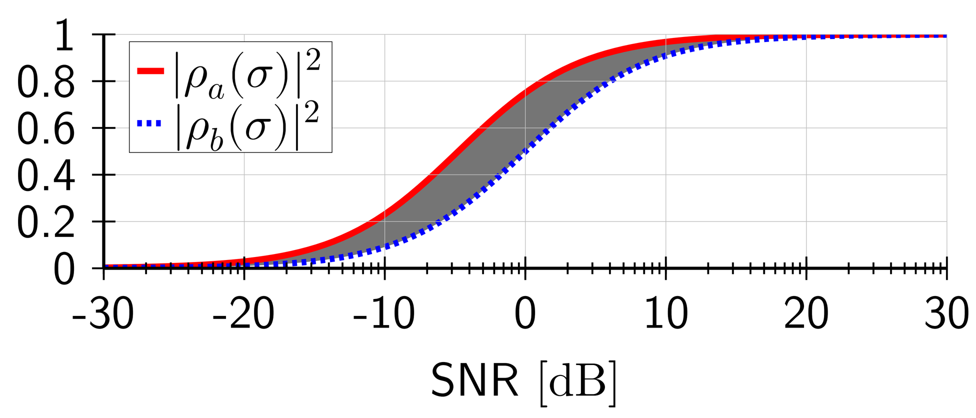


Figure 5. Squared spectral coherences in terms of SNR.

References

[1] J. Riba and M. Vila, “On infinite past predictability of cyclostationary signals,” *IEEE Signal Processing Letters*, vol. 29, pp. 647–651, 2022. DOI: 10.1109/lsp.2022.3149705.

[2] M. Vilà-Insa and J. Riba, “Asymptotic analysis of synchronous signal processing,” *preprint [arXiv:2403.18445]*, 2024. arXiv: 2403.18445 [eess.SP].

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