LOW-COMPLEXITY DETECTION OF PERMUTATIONAL INDEX MODULATION FOR NONCOHERENT COMMUNICATIONS

Signal Processing & Communications

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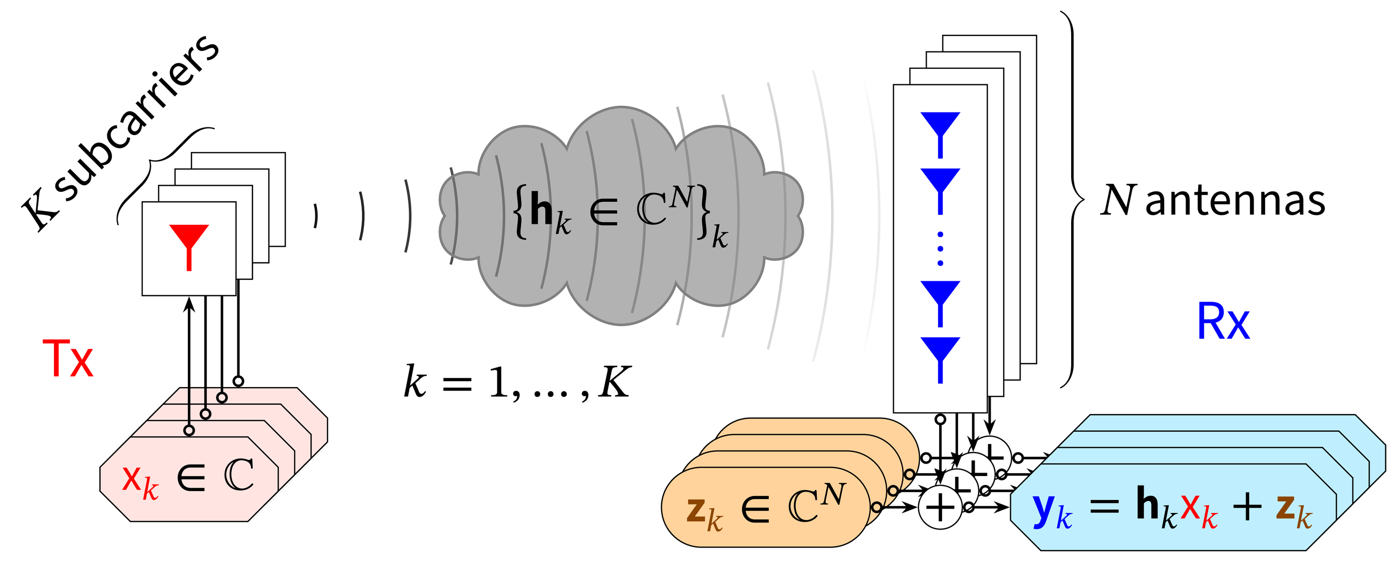
Highlights

- **Objective:** Design a massive SIMO scheme for wireless communications with one-shot noncoherent detection.
- **Approach:** Permutational index modulation over OFDM.
  - Convey information on the ordering in which a set of values is mapped onto OFDM subcarriers.
  - Spherical code with improved robustness against channel impairments.
  - Exploit statistical channel state information and hardening.
- **Outcome:** A simple detector based on sorting quadratic metrics of data.
  - Near-ML error performance.
  - Low-complexity implementation.

Background

- Next generation networks:
  - Accommodate a large number and variety of devices.
  - Heterogeneous and versatile systems.
  - Mainly constructed over OFDM.
- **Massive machine type communications (mMTC)**: small information volume with highly reliable transmission and severe latency constraints.
- **Proposal:** One-shot energy-based noncoherent detection [1].
  - No instantaneous channel state information (CSI) $\implies$ reduce training overhead.
  - Statistical CSI + massive SIMO $\implies$ channel hardening to achieve high reliability.
  - Multilevel index modulation (IM) $\implies$ increase spectral efficiency (SE).
  - Permutation modulation (PM) $\implies$ improve error performance of energy-based systems.
  - Sorting-based detection $\implies$ low-latency & low-complexity.

Problem statement



- Transmitted codewords $\mathbf{x} \triangleq [x_1, \dots, x_K]^{\mathrm{T}} \in \mathbb{C}^K$.
- Channel and noise are independent among different frequency bands.
- Statistical CSI: $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{C}_{\mathbf{h},k})$ and $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}_N, \mathrm{P_z}\mathbf{I}_N)$ known at Rx.
- Likelihood function:

No phase information

$$f_{\mathbf{y}|\mathbf{x}}(\{\mathbf{y}_k\}_k) = \prod_{k=1}^{K} \frac{\exp\left(-\mathbf{y}_k^{\mathrm{H}}\mathbf{C}_{\mathbf{y}|x,k}^{-1}\mathbf{y}_k\right)}{\pi^N |\mathbf{C}_{\mathbf{y}|x,k}|}, \quad \mathbf{C}_{\mathbf{y}|x,k} \triangleq |x_k|^2\mathbf{C}_{\mathbf{h},k} + \mathrm{P_z}\mathbf{I}_N \tag{1}$$

- Codebook $\mathcal{X}$: $\{x_k\}$ selected from L-ary unipolar pulse-amplitude modulation (PAM),

$$\mathcal{A}_L \triangleq \{|x_1| \triangleq 0 < |x_2| < \cdots < |x_L|\}, \quad L \geq 2. \tag{2}$$

Permutational index modulation (PIM)

- **PM:** Each codeword is constructed by permuting the elements of a reference vector

$$\underline{\mathbf{x}} \triangleq [\underline{x}_1, \dots, \underline{x}_K]^{\mathrm{T}} = [\underbrace{0, \dots, 0}_{K_1 \text{ (inactive)}}, \underbrace{|x_2|, \dots, |x_2|}_{K_2}, \dots, \underbrace{|x_L|, \dots, |x_L|}_{K_L}]^{\mathrm{T}}, \quad \textstyle\sum_{l=1}^{L} K_l = K. \tag{3}$$

- $\mathcal{X} = \Sigma_K(\underline{\mathbf{x}})$ (*i.e.* permutations of the reference vector), with $|\mathcal{X}| = K!/\prod_{l=1}^{L} K_l!$. Its spectral efficiency (SE) is maximized under uniform policy, *i.e.* $K_l = K/L$.
- For $K \to \infty$, the SE approaches $\mathrm{H}(\{K_l/K\}_l)$.

Maximum likelihood (ML) detection

- Pre-processing of the received signal: $\{\mathbf{r}_k \triangleq \mathbf{U}_k^{\mathrm{H}}\mathbf{y}_k\}_k$, with $\mathbf{C}_{\mathbf{h},k} \triangleq \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^{\mathrm{H}}$. Resulting distribution is $(\mathbf{r}_k|x_k) \sim \mathcal{CN}\left(\mathbf{0}_N, |x_k|^2\mathbf{\Lambda}_k + \mathrm{P_z}\mathbf{I}_N\right)$.
- ML detector: $\hat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\mathbf{x}\in\mathcal{X}} \sum_{k=1}^{K} \mathbf{r}_k^{\mathrm{H}}\left(|x_k|^2\mathbf{\Lambda}_k + \mathrm{P_z}\mathbf{I}_N\right)^{-1}\mathbf{r}_k + \ln\left||x_k|^2\mathbf{\Lambda}_k + \mathrm{P_z}\mathbf{I}_N\right|$.
- Complexity: $O(KN^2)$ (pre-processing) $+O(|\mathcal{X}|KN)$ (ML statistics).

References

[1] M. Vilà-Insa, A. Martí, M. Lamarca, and J. Riba, *Low-complexity detection of permutational index modulation for noncoherent communications*, 2025. arXiv: 2504.02946 [cs.IT].

[2] M. Vilà-Insa, A. Martí, J. Riba, and M. Lamarca, "Quadratic detection in noncoherent massive SIMO systems over correlated channels," *IEEE Trans. Wireless Commun.*, vol. 23, no. 10, pp. 14 259–14 272, 2024.

[3] M. Vilà-Insa and J. Riba, "Singular detection in noncoherent communications," *IEEE Wireless Commun. Lett.*, vol. 14, no. 4, pp. 1164–1168, 2025.

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Low complexity detection

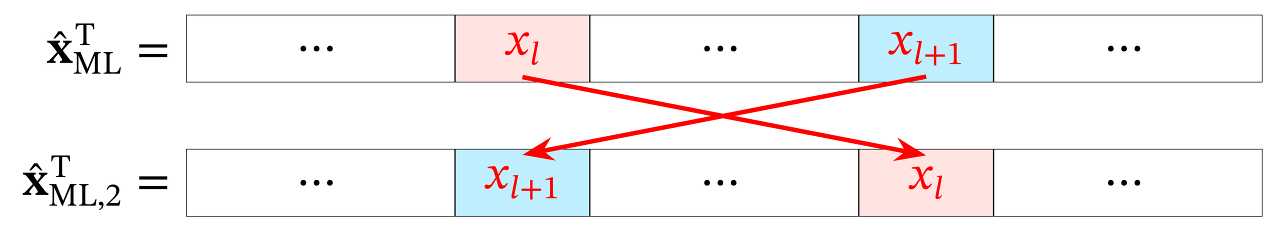
Uncorrelated fading

- The resulting ML detector simplifies to $\hat{\mathbf{x}}_{\mathrm{ML}} = \arg\min_{\sigma\in\Sigma_K} \sum_{k=1}^{K} u_k v_{\sigma(k)}$, where σ is an index permutation, $u_i \triangleq \|\mathbf{y}_i\|^2$ and $v_i \triangleq (|\underline{x}_i|^2 + \mathrm{P_z})^{-1}$.
- ML detection is equivalent to pairing the most energetic symbols with the most energetic received signal per band:

$$\varsigma = \underset{\sigma\in\Sigma_K}{\arg\,\mathrm{sort}}\{\|\mathbf{y}_k\|^2\} \implies \begin{array}{c} \left\{ \|\mathbf{y}_{\varsigma(1)}\|^2 < \|\mathbf{y}_{\varsigma(2)}\|^2 < \dots < \|\mathbf{y}_{\varsigma(K)}\|^2 \right\} \\ \Updownarrow \quad \Updownarrow \quad \Updownarrow \\ \hat{\mathbf{x}}_{\mathrm{ML}}^{\mathrm{T}} = \left[\underline{x}_{\varsigma^{-1}(1)}, \underline{x}_{\varsigma^{-1}(2)}, \dots, \underline{x}_{\varsigma^{-1}(K)} \right] \end{array}. \tag{4}$$

- Complexity: $O(KN) + O(K\ln K)$.

Likelihood ratio:



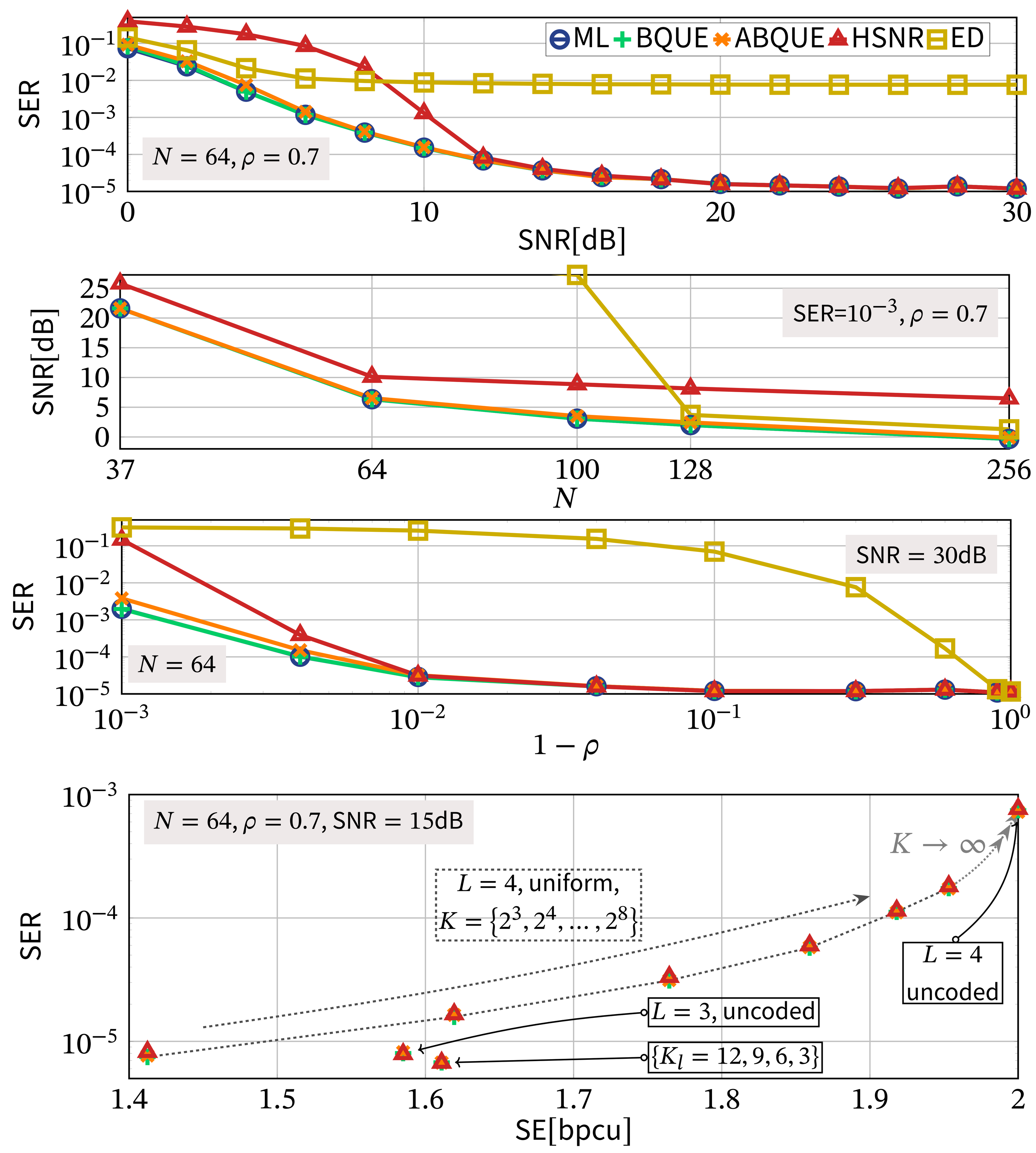
Proposed scheme

- Energy estimators developed in [2]: $\hat{\beta}_k(\mathbf{r}_k) \triangleq \mathbf{r}_k^{\mathrm{H}}\mathbf{A}_k\mathbf{r}_k - \mathrm{Tr}(\mathbf{A}_k)$.
- Low-complexity detection: $\tilde{\varsigma} = \arg\,\mathrm{sort}_{\sigma\in\Sigma_K}\{\hat{\beta}_k(\mathbf{r}_k)\}$.
- Complexity: $O(KN^2) + O(KN) + O(K\ln K)$.
- Three designs:
  1. *Energy detector (ED):* $\mathbf{A}_{\mathrm{ED},k} \triangleq \mathbf{I}_N / \mathrm{Tr}(\mathbf{\Lambda}_k)$.
  2. *High SNR (HSNR):* $\mathbf{A}_{\mathrm{HSNR},k} \triangleq \mathbf{\Lambda}_k^{-1}/N$.
  3. *Assisted best quadratic unbiased estimator (ABQUE):*

$$\hat{\mathbf{x}}_{\mathrm{ED}} \longrightarrow \mathbf{A}_{\mathrm{ABQUE},k} \triangleq \mathbf{\Lambda}_k\mathbf{C}_{\mathbf{r}|\hat{x},k}^{-2} \,/\, \|\mathbf{\Lambda}_k\mathbf{C}_{\mathbf{r}|\hat{x},k}^{-1}\|_{\mathrm{F}}^2 \longrightarrow \hat{\mathbf{x}}_{\mathrm{ABQUE}}. \tag{5}$$

Numerical results

- Default configuration: $K = 32$, $L = 4$, uniform policy, 4-ASK, $[\mathbf{C_h}]_{m,n} = \rho^{|n-m|}$.
- Fundamental error floor at high SNR [3].



Future work

- The use of tailored constellations [4] and optimized nonuniform partitions are expected to yield better performance.
- Alphabet trimming to further improve its robustness to detection errors.
- Combined coherent-noncoherent schemes for multi-resolution systems.

Funding

This work was funded by project MAYTE (PID2022-136512OB-C21) by MICIU/AEI/10.13039/501100011033 and ERDF/EU, grants 2021 SGR 01033 and 2022 FI SDUR 00164 by Departament de Recerca i Universitats de la Generalitat de Catalunya, and grant 2023 FI "Joan Oró" 00050 by Departament de Recerca i Universitats de la Generalitat de Catalunya and the ESF+.

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