

Summary

What the bottleneck is:

- Rectangular-pulse CP-OFDM has slow sidelobe decay ($\propto f^{-2}$), leading to high OBR/ACI.
- Practical designs must jointly provide: (i) tight OBR control; (ii) transmit-power budget; (iii) reliable detection over frequency-selective channels.
- Most spectral precoders are CSIT-blind; PSP-OFDM [1] is an exception.

What we propose:

- A CSIT-aware BSPM precoder that minimizes OBR in band $\mathcal{B}$ under transmit-power and MI constraints.
- Solved by a fixed-point algorithm combining reverse waterfilling and a semi-unitary front-end.
- Spectral invariance under unitary rotations embeds CSIT without altering the transmit PSD/OBR.
- Enables low-complexity ZF (SVD) and SIC (GMD) receivers.

Proposed Design

Fixed-point precoder algorithm

Input: initial $\mathbf{G}_0$, $\mu_0 > 0$.

Repeat for $i = 1, 2, \dots$ until convergence:

1. *Fixed-point update:*

$$\tilde{\mathbf{G}}_i = (\mathbf{A} + \mu_{i-1} \mathbf{I}_K)^{-1} \mathbf{C} \mathbf{G}_{i-1} (\mathbf{I}_D + \mathbf{G}_{i-1}^H \mathbf{C} \mathbf{G}_{i-1})^{-1}.$$

2. *MI scaling:* find $\alpha_i \geq 0$ such that $\log |\mathbf{I}_D + \alpha_i^2 \tilde{\mathbf{G}}_i^H \mathbf{C} \tilde{\mathbf{G}}_i| = \mathcal{I}_{\min}$.

3. *Precoder update:* $\mathbf{G}_i \leftarrow \alpha_i \tilde{\mathbf{G}}_i$.

4. *Power-multiplier update:* $\mu_i \leftarrow \mu_{i-1} \cdot \frac{\text{Tr}(\mathbf{G}_i^H \mathbf{G}_i)}{P_{\max}}$.

Output: optimized precoder $\mathbf{G}_\star$ at convergence.

Unitary degree of freedom:

- Let $M = \text{rank}(\mathbf{G}_\star) \leq D$. If $M < D$, retain the active modes of $\mathbf{G}_\star$.
- For any unitary $\mathbf{V} \in \mathbb{C}^{M \times M}$, $\mathbf{G}_\star \mathbf{V}$ preserves the PSD/OBR and can be chosen to shape the effective channel $\mathbf{H} \mathbf{G}_\star \mathbf{V}$ for ZF/SIC decoding.

System Overview

• **Multicarrier setup:** N -point IFFT system with $K \leq N$ active subcarriers; guard interval l_g .

Spectral precoding:

- Map $D \leq K$ symbols $\mathbf{d} \in \mathbb{A}^D$ to the active tones: $\mathbf{x} = \mathbf{G} \mathbf{d} \in \mathbb{C}^K$
- $\mathbf{G} \in \mathbb{C}^{K \times D}$: redundancy $R = K - D$, rate $\eta = D/K \leq 1$.

PSD structure:

- Under $\mathbb{E}[\mathbf{d}_m \mathbf{d}_{m-\ell}^H] = \delta[\ell] \mathbf{I}_D$, the PSD expression has the form [2]:

$$P_s(f) \propto \mathbf{a}^H(f) \mathbf{G} \mathbf{G}^H \mathbf{a}(f).$$

- $\mathbf{a}(f)$: stacked per-subcarrier spectral shapes.
- PSD depends on $\mathbf{G}$ through $\mathbf{G} \mathbf{G}^H$ (invariant to right-unitary rotations).

Channel:

- Under standard CP-OFDM, the post-FFT channel is diagonal:

$$\mathbf{H} = \text{diag} \{H_c[k_1], \dots, H_c[k_K]\} \in \mathbb{C}^{K \times K}.$$

- $H_c[k]$ is the N -point DFT sample of the channel impulse response.

Received signal:

- The received vector (after CP removal and FFT):

$$\mathbf{r} = \mathbf{H} \mathbf{G} \mathbf{d} + \mathbf{n}, \quad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_K).$$

Optimization Framework

Three quantities at play:

- OBR** in the target band $\mathcal{B}$:

$$P_W = \text{Tr}(\mathbf{G}^H \mathbf{A} \mathbf{G}), \quad \mathbf{A} \succeq \mathbf{0}.$$

with $\mathbf{A}$ captures leakage weighted by $W(f)$ in $\mathcal{B}$.

- Transmit power:** $\mathbb{E}[\|\mathbf{x}\|^2] = \text{Tr}(\mathbf{G}^H \mathbf{G})$.

- MI:** $\mathcal{I}(\mathbf{d}; \mathbf{r}) = \log |\mathbf{I}_D + \mathbf{G}^H \mathbf{C} \mathbf{G}|$, $\mathbf{C} \triangleq \frac{1}{\sigma^2} \mathbf{H}^H \mathbf{H}$.

Design problem:

$$\min_{\mathbf{G}} \text{Tr}(\mathbf{G}^H \mathbf{A} \mathbf{G}) \quad \text{s.t.} \quad \text{Tr}(\mathbf{G}^H \mathbf{G}) \leq P_{\max}, \log |\mathbf{I}_D + \mathbf{G}^H \mathbf{C} \mathbf{G}| \geq \mathcal{I}_{\min}.$$

Key properties:

- The problem is non-convex.
- The MI constraint is tight at the optimum.
- Feasibility can be checked by waterfilling: $\mathcal{I}_{\min} \leq \mathcal{I}_0$, or equivalently by reverse waterfilling: $P_{\max} \geq P_0$.

First-order (KKT) condition:

$$(\mathbf{A} + \mu \mathbf{I}_K) \mathbf{G} = \alpha \mathbf{C} \mathbf{G} (\mathbf{I}_D + \mathbf{G}^H \mathbf{C} \mathbf{G})^{-1}, \quad \mu, \alpha \geq 0.$$

- This condition motivates the proposed fixed-point iteration.

Solution insight:

- For each fixed $\mu \geq 0$, find $(\mathbf{G}_\mu, \alpha_\mu)$ satisfying KKT with the MI constraint tight.
- Adapt μ to drive the transmit power to $P_{\max}$.
- Conjecture:** as $\mu \uparrow$, $\text{Tr}(\mathbf{G}_\mu^H \mathbf{G}_\mu) \downarrow$ and $\text{Tr}(\mathbf{G}_\mu^H \mathbf{A} \mathbf{G}_\mu) \uparrow$.

Decoder-Aware Design

ZF receiver:

- Apply SVD: $\mathbf{H} \mathbf{G}_\star = \mathbf{U}_h \mathbf{\Gamma}_h \mathbf{V}_h^H$, $\mathbf{\Gamma}_h = \text{diag} \{\gamma_1, \dots, \gamma_M\}$.
- Transmit: $\mathbf{x} = \mathbf{G}_\star \mathbf{V}_h \mathbf{d} \Rightarrow \mathbf{r} = \mathbf{U}_h \mathbf{\Gamma}_h \mathbf{d} + \mathbf{n}$.
- ZF detection: $\hat{\mathbf{d}} = \text{DEC}\{\mathbf{\Gamma}_h^{-1} \mathbf{U}_h^H \mathbf{r}\}$, $\mathbf{w}_h \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{\Gamma}_h^{-2})$.

SIC receiver:

- Apply GMD: $\mathbf{H} \mathbf{G}_\star = \gamma \mathbf{Q}_h \mathbf{R}_h \mathbf{P}_h^H$, $\gamma = \left(\prod_{i=1}^M \gamma_i\right)^{1/M}$, where $\mathbf{R}_h$ is unit upper triangular.
- Transmit: $\mathbf{x} = \mathbf{G}_\star \mathbf{P}_h \mathbf{d}$.
- After projection: $\tilde{\mathbf{r}} \triangleq \gamma^{-1} \mathbf{Q}_h^H \mathbf{r} = \mathbf{R}_h \mathbf{d} + \tilde{\mathbf{w}}_h$, where $\tilde{\mathbf{w}}_h \sim \mathcal{CN}(\mathbf{0}, \frac{\sigma^2}{\gamma^2} \mathbf{I}_M)$.
- Backward SIC: $\hat{d}_M = \text{DEC}\{\tilde{r}_M\}$, $\hat{d}_\tau = \text{DEC}\{\tilde{r}_\tau - \sum_{\nu>\tau} \rho_{\tau\nu} \hat{d}_\nu\}$, $\tau = M-1, \dots, 1$.

- Main benefit:** GMD-SIC equalizes the post-processing noise variance and avoids weak-mode ZF noise enhancement, with no PSD/OBR penalty.

Numerical Results

Simulation setup:

- Waveform & channel:** CP-OFDM, rect. pulse, ideal LP DAC; $N = 128$, $l_g = 16$, $K = 65$ contiguous tones; flat $W(f)$ on $\mathcal{B}$; block-fading exp. PDP factor $\delta = 0.1$, $\sum_{k \in \mathcal{K}} |H_c[k]|^2 = K$, 16-QAM modulation.
- Constraints:** $P_{\max} = D$, $\mathcal{I}_{\min} = \zeta \mathcal{I}_0$, $\zeta \in (0, 1)$; 10^3 ch. realizations.

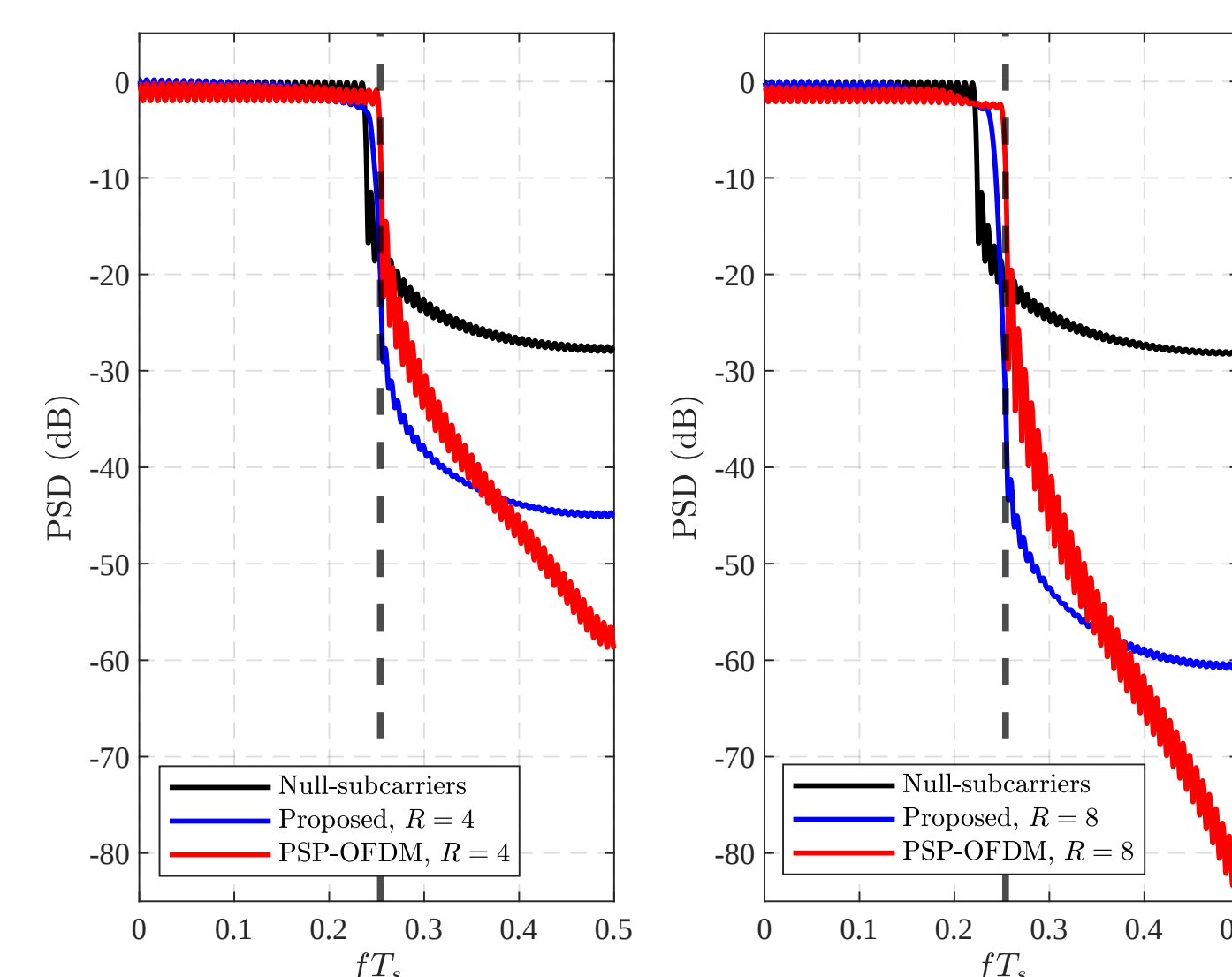


Fig. 1. PSDs of the prop. & PSP-OFDM precoders with $R \in \{4, 8\}$, $\zeta = 0.99$.

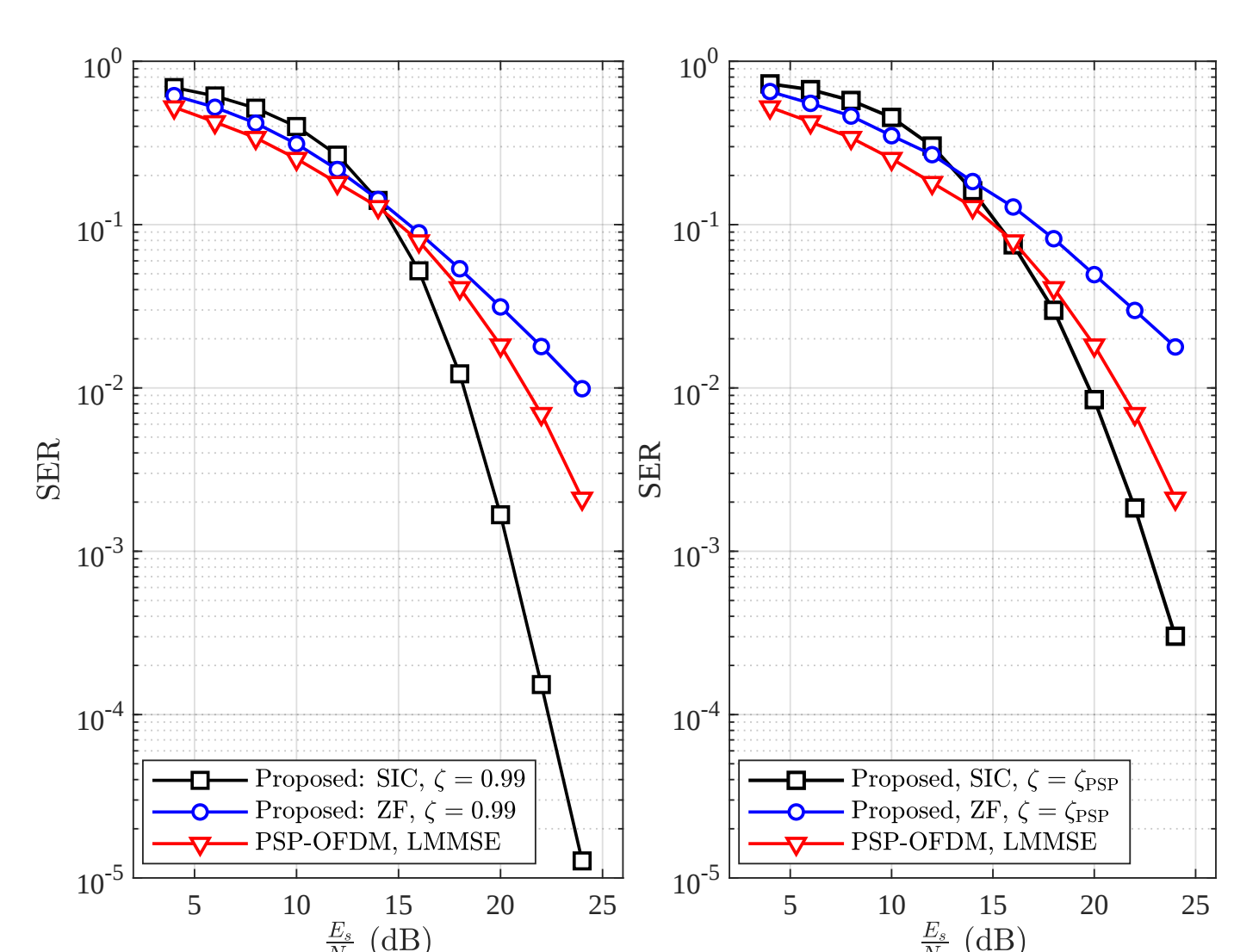


Fig. 2. SER of the prop. & PSP-OFDM precoders, with $R=6$, $\zeta \in \{0.99, \zeta_{\text{PSP}}\}$.

ζ	0.95	0.96	0.97	0.98	0.99	0.995
OBR	19.79	23.51	27.11	30.60	31.78	34
OBR red.	29.59	25.87	22.27	19.22	17.61	15.38

Table 1. Effect of MI: OBR and OBR redu. relative to plain OFDM (in dB).

References & Acknowledgment

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