Channel-Aware Orthogonal Precoder for Sidelobe Suppression in OFDM

atlanTTic research center for Telecommunication Technologies
Universida de Vigo

Vahid Vahidpour[1], Roberto López-Valcarce[1], and Josep Sala-Álvarez[2]
[1]atlanTTic Research Center, Universidade de Vigo, {vahidpour,valcarce}@gts.uvigo.es
[2]Universitat Politècnica de Catalunya, josep.sala@upc.edu

UPC
UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH
Departament de Teoria del Senyal i Comunicacions

Summary

- **What we propose:**
  - A channel-aware semi-unitary precoder that minimizes OBR over $\mathcal{B}$.
  - The spectral front-end is obtained from the eigenvectors associated with the smallest eigenvalues of the OBR matrix $\mathbf{\Phi}$.
  - Right-unitary invariance embeds CSIT without changing the transmit PSD/OBR.
  - SVD- and GMD-based rotations enable low-complexity ZF and SIC receivers, respectively.

System Overview

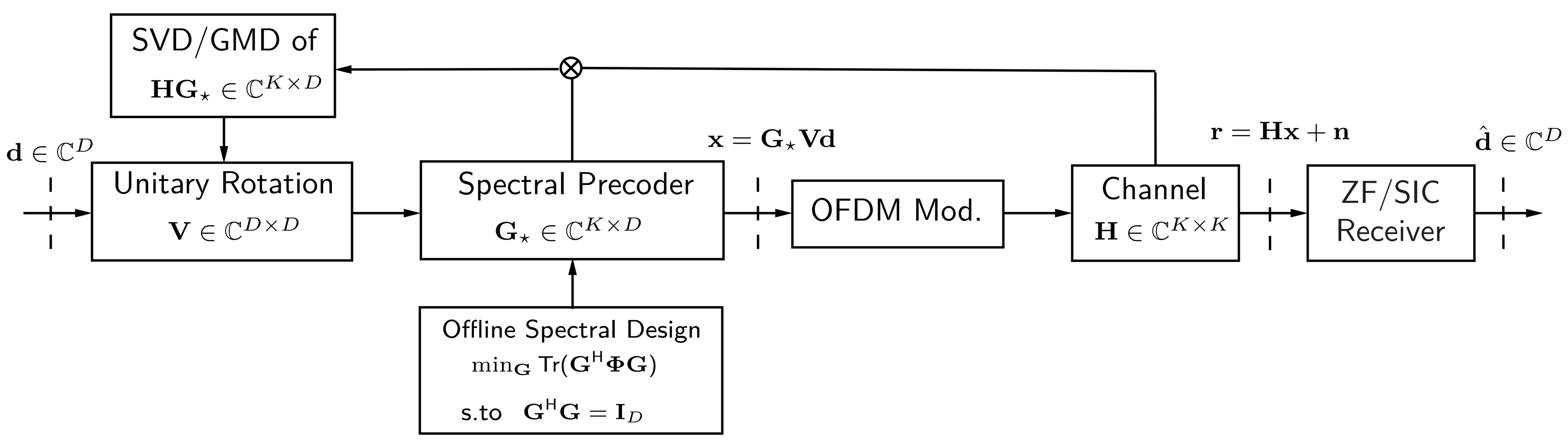

Fig.1 Proposed Channel-Aware Spectral Precoding Chain.

- **Spectral precoding:**
  - Map $D\le K$ symbols $\mathbf{d}\in\mathbb{A}^D$ onto the K active tones: $\mathbf{x}=\mathbf{Gd}\in\mathbb{C}^K$
  - $\mathbf{G}\in\mathbb{C}^{K\times D}$: with redundancy $R=K-D$, coding rate $\lambda=D/K\le 1$.
- **PSD structure:**
  - Under $\mathsf{E}[\mathbf{d}_m\mathbf{d}^{\mathsf H}_{m-\ell}]=\delta[\ell]\mathbf{I}_D$, the continuous-time PSD has the form [1]:

$$\boxed{P_s(f)\ \propto\ \boldsymbol{\phi}^{\mathsf H}(f)\,\mathbf{GG}^{\mathsf H}\,\boldsymbol{\phi}(f).}$$

  - $\boldsymbol{\phi}(f)\in\mathbb{C}^K$: stacks the spectral shapes of the active subcarriers.
  - PSD depends on $\mathbf{G}$ through $\mathbf{GG}^{\mathsf H}$ (invariant to right-unitary rotations).
- **Channel:**
  - Under standard CP-OFDM, the post-FFT channel is diagonal:

$$\mathbf{H}=\mathrm{diag}\left\{H_c[k_1],\dots,H_c[k_K]\right\}\in\mathbb{C}^{K\times K}.$$

  - $H_c[k]$: N-point DFT sample of the channel impulse response at k.
- **Received signal:**
  - The received vector (after CP removal and N-point FFT):

$$\mathbf{r}=\mathbf{HGd}+\mathbf{n},\quad \mathbf{n}\sim\mathcal{CN}(\mathbf{0},\sigma^2\mathbf{I}_K).$$

Optimization Framework

- **Problem statement:**
  - Define the weighted transmit power over the protected band $\mathcal{B}$

$$P_W\triangleq\int_{-\infty}^{\infty}W(f)P_s(f)df=\mathrm{Tr}(\mathbf{G}^{\mathsf H}\mathbf{\Phi}\mathbf{G}),\quad \mathbf{\Phi}\succeq\mathbf{0}.$$

  - $\mathbf{\Phi}\in\mathbb{C}^{K\times K}$ captures leakage weighted by $W(f)\ge 0$ in $\mathcal{B}$.
  - To avoid the trivial solution $\mathbf{G}=\mathbf{0}$, enforce a semi-unitary constraint:

$$\boxed{\min_{\mathbf{G}}\ \mathrm{Tr}(\mathbf{G}^{\mathsf H}\mathbf{\Phi}\mathbf{G})\quad \text{s.t.}\quad \mathbf{G}^{\mathsf H}\mathbf{G}=\mathbf{I}_D}$$

- **Closed-form solution:**
  - Consider the eigendecomposition

$$\mathbf{\Phi}=\mathbf{Z}\mathrm{diag}\left\{\nu_1,\dots,\nu_K\right\}\mathbf{Z}^{\mathsf H},\qquad \nu_1\ge\nu_2\ge\cdots\ge\nu_K\ge 0$$

  where $\mathbf{Z}=[\mathbf{z}_1,\dots,\mathbf{z}_K]$ is unitary.
  - One optimal semi-unitary precoder is

$$\boxed{\mathbf{G}_\star=[\,\mathbf{z}_{K-D+1},\mathbf{z}_{K-D+2},\dots,\mathbf{z}_K\,]}$$

  - Hence, $\mathbf{G}_\star$ spans the eigenspace associated with the D smallest eigenvalues of $\mathbf{\Phi}$.
  - $\mathbf{G}_\star$ is unique up to a right-unitary rotation: $\mathbf{G}_\star\mathbf{V}$ is spectrally equivalent, $\mathbf{V}\in\mathbb{C}^{D\times D}$ is free for CSIT, at zero spectral cost.

Proposed Precoder Design

- **ZF without CSIT** *(baseline)*:
  - Transmit: $\mathbf{x}=\mathbf{G}_\star\mathbf{d}\quad\Rightarrow\quad\mathbf{r}=\mathbf{HG}_\star\mathbf{d}+\mathbf{n}.$
  - ZF detection: $\hat{\mathbf{d}}=\mathsf{DEC}\{\mathbf{G}_\star^{\mathsf H}\mathbf{H}^{-1}\mathbf{r}\}$, $\mathbf{w}\sim\mathcal{CN}(\mathbf{0},\sigma^2\mathbf{G}_\star^{\mathsf H}(\mathbf{H}^{\mathsf H}\mathbf{H})^{-1}\mathbf{G}_\star)$.
- **ZF with CSIT:**
  - Apply SVD: $\mathbf{HG}_\star=\mathbf{U}_c\mathbf{\Gamma}_c\mathbf{V}_c^{\mathsf H}$, $\quad\mathbf{\Gamma}_c=\mathrm{diag}\left\{\gamma_1,\dots,\gamma_D\right\}$.
  - Transmit: $\mathbf{x}=\mathbf{G}_\star\mathbf{V}_c\mathbf{d}\quad\Rightarrow\quad\mathbf{r}=\mathbf{U}_c\mathbf{\Gamma}_c\mathbf{d}+\mathbf{n}.$
  - ZF detection: $\hat{\mathbf{d}}=\mathsf{DEC}\{\mathbf{\Gamma}_c^{-1}\mathbf{U}_c^{\mathsf H}\mathbf{r}\}$, $\quad\mathbf{w}_c\sim\mathcal{CN}\left(\mathbf{0},\sigma^2\mathbf{\Gamma}_c^{-2}\right)$.
- **SIC with CSIT:**
  - Apply GMD: $\mathbf{HG}_\star=\gamma\mathbf{Q}_c\mathbf{R}_c\mathbf{P}_c^{\mathsf H}$, $\quad\gamma=\left(\prod_{i=1}^{D}\gamma_i\right)^{1/D}$, where $\mathbf{R}_c$ is unit upper triangular and $\mathbf{Q}_c$, $\mathbf{P}_c$ (semi)-unitary.
  - Transmit: $\mathbf{x}=\mathbf{G}_\star\mathbf{P}_c\mathbf{d}$.
  - After projection: $\tilde{\mathbf{r}}\triangleq\gamma^{-1}\mathbf{Q}_c^{\mathsf H}\mathbf{r}=\mathbf{R}_c\mathbf{d}+\tilde{\mathbf{w}}_c$, where $\tilde{\mathbf{w}}_c\sim\mathcal{CN}\left(\mathbf{0},\frac{\sigma^2}{\gamma^2}\mathbf{I}_D\right)$.
  - Backward SIC:

$$\hat{d}_D=\mathsf{DEC}\{\tilde{r}_D\},\quad \hat{d}_\tau=\mathsf{DEC}\{\tilde{r}_\tau-\sum_{\nu>\tau}\rho_{\tau\nu}\hat{d}_\nu\},\quad \tau=D-1,\dots,1.$$

Numerical Results

- **Simulation setup:**
  - **Waveform & channel:** CP-OFDM, rect. pulse, ideal LP DAC; $N=256$, $l_g=32$, $K=129$ contiguous tones; flat $W(f)$ on $\mathcal{B}$; block-fading exp. PDP factor $\delta=0.1$, $\sum_{k\in\mathcal{K}}|H_c[k]|^2=K$, 16-QAM modulation.
- **Sidelobe suppression:**

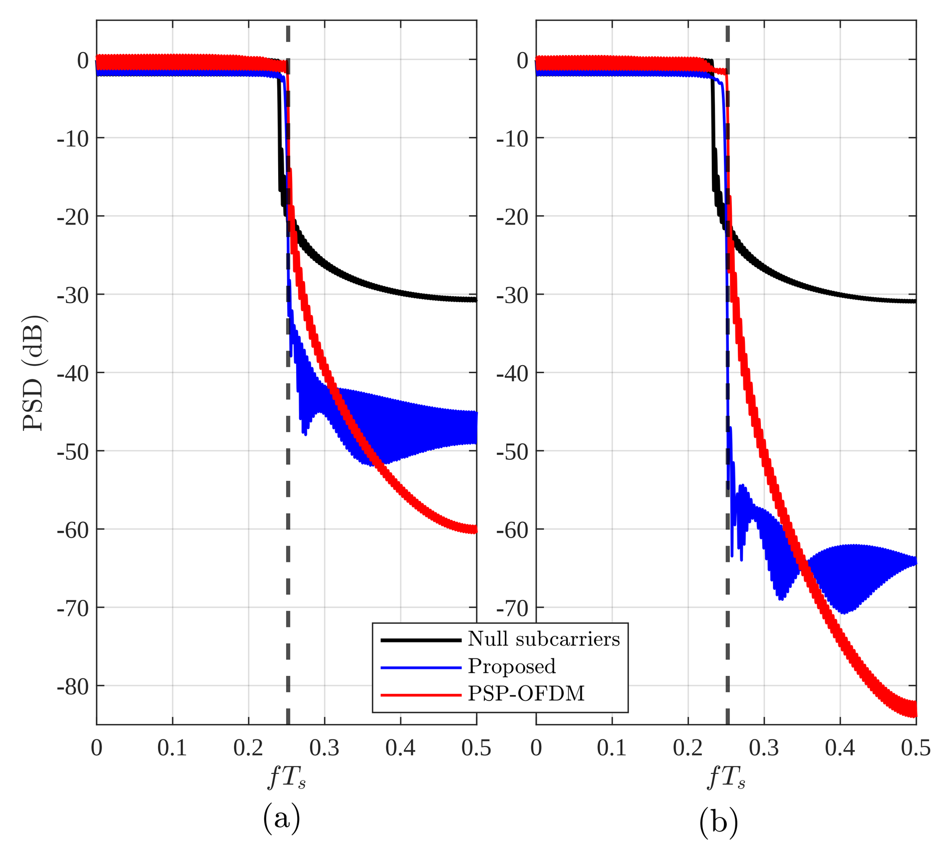

Fig. 2: PSDs of proposed and PSP-OFDM [2] precoders ($\delta=0.10$); with $R=6$ (left), and $R=10$ (right).

- **Error rate:**

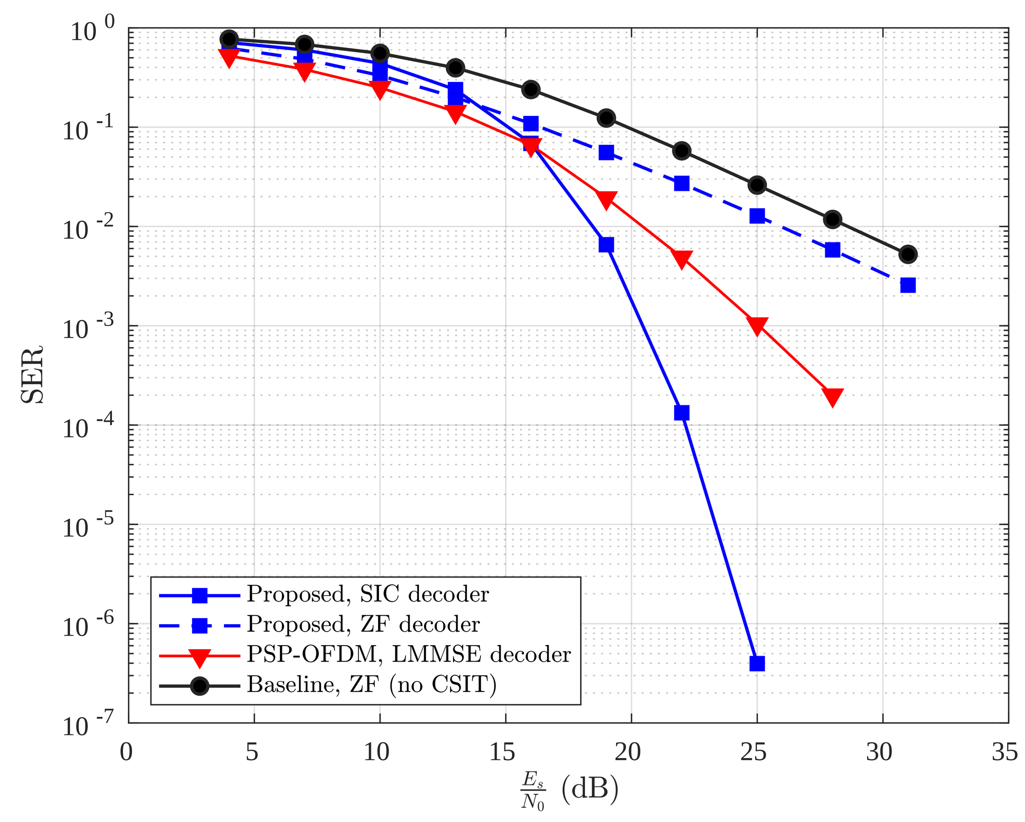

Fig. 3: SER performance of proposed and PSP-OFDM [2] precoders (ZF and SIC), with $\delta=0.10$ and $R=6$.

References & Acknowledgment

[1] R. López-Valcarce, "General form of the power spectral density of multicarrier signals," *IEEE Commun. Lett.*, vol. 26, no. 8, pp. 1755–1759, 2022.

[2] W.-C. Chen, C.-D. Chung, and P.-H. Wang, "Pre-equalized and spectrally precoded OFDM," *IEEE Trans. Veh. Technol.*, vol. 71, no. 7, pp. 7472–7486, 2022.

Work under project MAYTE (PID2022-136512OB-C22 and PID2022-136512OB-C21, financed by MICIU/AEI/10.13039/501100011033 and by ERDF/EU) :

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN