Multibeam Analog Beamformer Design for Monostatic ISAC under Self-Interference

Jordi Borras and **Roberto López-Valcarce**

2026 IEEE International Conference on Acoustics, Speech, and Signal Processing

Barcelona, Spain

May 7th, 2026

Grants PID2022-136512OB-C21 & PID2022-136512OB-C22 funded by:

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

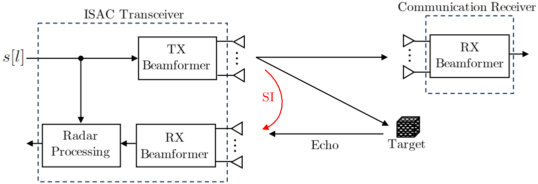
Cofinanciado por la Unión Europea

AGENCIA ESTATAL DE INVESTIGACIÓN

ISAC in mmWave Networks

- **The ISAC paradigm**
  - Sharing the hardware platform, signaling, and radio spectrum.
  - A core enabler for 6G applications (V2X, UAVs).
- **Suitability of mmWave**
  - Offers high angular and range resolution.
  - Demands directional beamforming to overcome severe path loss.
- **The Hardware Reality**
  - Fully digital architectures are often prohibitive in power and cost.
  - Analog beamforming (single RF chain) imposes strict Constant Modulus (CM) constraints.

The Challenge: Monostatic Operation & Self-Interference

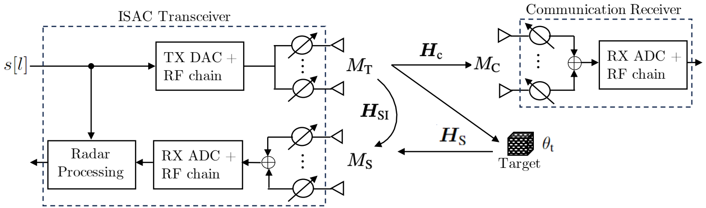


- **The Self-Interference (SI) Bottleneck**
  - The powerful TX signal strongly leaks into the sensing RX array.
- **Hardware Vulnerability & RX Cancellation Limits**
  - RF cancellation at RX is useful, but cannot protect RF components preceding the canceller.
  - Early component saturation ruins any subsequent analog/digital cancellation.

Our Approach: SI-aware TX Beamforming

Design a TX analog beamformer that maximizes ISAC performance while proactively limiting the SI level at the RX sensing array.

System Model (I)



- Beamformers: $\boldsymbol{f} \in \mathbb{V}^{M_\mathsf{T}}$ (TX); $\boldsymbol{w}_\mathsf{s} \in \mathbb{V}^{M_\mathsf{S}}$ (Radar); $\boldsymbol{w}_\mathsf{c} \in \mathbb{V}^{M_\mathsf{C}}$ (RX)

$$\mathbb{V}^m \triangleq \{\boldsymbol{z} \in \mathbb{C}^m \ : \ |z_i| = 1,\ 1 \leq i \leq m\}$$

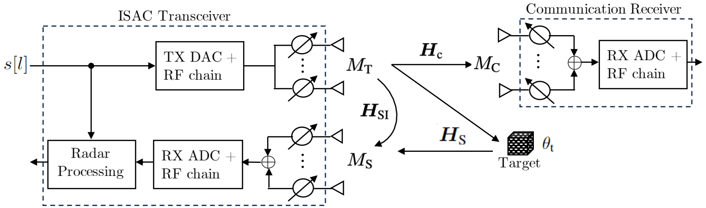
- Each antenna is connected to a PS without individual gain control
- ISAC transceiver sends zero-mean, unit-variance symbols:

$$\boldsymbol{x}[l] = \boldsymbol{f} s[l] \in \mathbb{C}^{M_\mathsf{T}}$$

- Received baseband communication signal:

$$y[l] = \boldsymbol{w}_\mathsf{c}^H \boldsymbol{H}_\mathsf{c} \boldsymbol{f} s[l] + \boldsymbol{w}_\mathsf{c}^H \boldsymbol{n}_\mathsf{c}[l] \xrightarrow{\text{for } \boldsymbol{f} \text{ given}} [\boldsymbol{w}_\mathsf{c}]_i = e^{\imath \measuredangle [\boldsymbol{H}_\mathsf{c} \boldsymbol{f}]_i},\ \forall i$$

System Model (and II)



- The radar observation is given by

$$r[l] = \underbrace{\boldsymbol{w}_{\mathsf{s}}^H \boldsymbol{H}_{\mathsf{S}} \boldsymbol{f} s[l]}_{\text{Target's Echo}} + \underbrace{\boldsymbol{w}_{\mathsf{s}}^H \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f} s[l]}_{\text{Post-combining SI}} + \underbrace{\boldsymbol{w}_{\mathsf{s}}^H \boldsymbol{n}_{\mathsf{s}}[l]}_{\text{Noise}}$$

- Conventional approach: mitigate Post-Combining SI via $\{\boldsymbol{f}, \boldsymbol{w}_{\mathsf{s}}\}$

Our Approach: Control *Pre-Combining* SI

$$P_{\mathsf{SI}} \triangleq \mathbb{E}\{\|\boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f} s[l]\|^2\} = \boldsymbol{f}^H \boldsymbol{H}_{\mathsf{SI}}^H \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f}$$

Problem Formulation

Multibeam SI-aware TX Beamforming

The design of $\boldsymbol{f}$ can be addressed as

$$\max_{\boldsymbol{f}} \; G_{\mathsf{tx,comm}} \triangleq \boldsymbol{f}^H \boldsymbol{H}_{\mathsf{c}}^H \boldsymbol{H}_{\mathsf{c}} \boldsymbol{f}$$

subject to:

- Required Beampattern gain: $G_{\mathsf{tx,sen}}(\theta_{\mathsf{t}}) \triangleq \left| \boldsymbol{f}^H \boldsymbol{a}_{\mathsf{T}}(\theta_{\mathsf{t}}) \right|^2 \geq \tau^2$
- Controlled Pre-combining SI: $P_{\mathsf{SI}} \triangleq \boldsymbol{f}^H \boldsymbol{H}_{\mathsf{SI}}^H \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f} \leq \eta^2$
- Constant-Modulus entries: $\boldsymbol{f} \in \mathbb{V}^{M_{\mathsf{T}}} \implies |f_i| = 1, \; i \in \{1, \ldots, M_{\mathsf{T}}\}$

Straightforward Approaches

Direct Projection

- $\widetilde{\boldsymbol{f}}_\star$: relaxed solution obtained by ~~$\boldsymbol{f} \in \mathbb{V}^{M_\mathsf{T}}$~~ $\rightarrow \|\boldsymbol{f}\|^2 = M_\mathsf{T}$
- The CM solution reads as

$$\boldsymbol{f}_\star = \mathcal{P}_{\mathbb{V}^n}\{\widetilde{\boldsymbol{f}}_\star\}$$

where $\mathcal{P}_{\mathbb{V}^n}\{\boldsymbol{x}\} \ : \ \mathbb{C}^n \rightarrow \mathbb{V}^n$, so that $\mathcal{P}_{\mathbb{V}^n}\{\boldsymbol{x}\} = \left[\frac{x_1}{|x_1|}, \ldots, \frac{x_n}{|x_n|}\right]^T$

QCQP+SDR

- Square up CM constraints:

$$|f_i| = 1 \rightarrow |f_i|^2 = 1 \iff \boldsymbol{f}^H \boldsymbol{e}_i \boldsymbol{e}_i^H \boldsymbol{f} = 1, \ \forall i$$

- Quadratically-Constrained Quadratic Problem (QCQP) with $M_\mathsf{T} + 2$ constraints

  $\rightarrow$ SDR (generally not tight)

Proposed Approach

SI-Aware Analog Multibeam Design

1. Obtain a Semi-Analytic Solution by relaxing CM constraints
2. Modify the Relaxed Solution to handle CM constraints

Semi-Analytic Relaxed Solution

- Replacing CM constraints by a Total Power Constraint, we have

$$\max_{\boldsymbol{f}\in\mathbb{C}^{M_\mathsf{T}}} \quad G_{\mathsf{tx,comm}}$$
$$\text{s.t.} \quad \text{(a) } G_{\mathsf{tx,sen}}(\theta_\mathsf{t}) \geq \tau^2, \text{ (b) } P_{\mathsf{SI}} \leq \eta^2, \text{ (c) } \cancel{\boldsymbol{f}\in\mathbb{V}^{M_\mathsf{T}}} \rightarrow \|\boldsymbol{f}\|^2 = M_\mathsf{T}$$

$\rightarrow$ QCQP with 3 constraints

- Structure of the *Relaxed* Optimal Beamformer:

$$\boldsymbol{f}_\star = \sqrt{M_\mathsf{T}}\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \mu_\star\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \gamma_\star\boldsymbol{H}_{\mathsf{SI}}^H\boldsymbol{H}_{\mathsf{SI}}\right]$$

$\rightarrow$ $\mathcal{D}[\boldsymbol{X}]$: dominant unit-norm eigenvector of $\boldsymbol{X}$
$\rightarrow$ $\mu_\star$, $\gamma_\star$: optimal Lagrange multipliers for (a) and (b)

$\mu_\star$ and $\gamma_\star$ do not admit closed-form solution $\rightarrow$ 3-step procedure

Relaxed Optimal Beamformer

3-step Procedure

1. **Communication-Optimal Start** $(\mu = \gamma = 0)$
   - → Initialize with $\boldsymbol{f} = \sqrt{M_\mathsf{T}}\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H \boldsymbol{H}_\mathsf{c}\right]$
   - → If feasible, stop.

2. **Single-Constraint Activation** $(\mu = 0$ or $\gamma = 0)$
   - → *Communication-Sensing beamformer* $\boldsymbol{f}_{\text{C-Sen}}$: set $\gamma = 0$, find μ
   - → *Communication-SIC beamformer* $\boldsymbol{f}_{\text{C-SIC}}$: set $\mu = 0$, find γ
   - → Bisection Search

$$\boldsymbol{f}_\star = \begin{cases} \boldsymbol{f}_{\text{C-Sen}} & \text{if only } \boldsymbol{f}_{\text{C-Sen}} \text{ is feasible} \\ \boldsymbol{f}_{\text{C-SIC}} & \text{if only } \boldsymbol{f}_{\text{C-SIC}} \text{ is feasible} \\ \arg\max_{\{\boldsymbol{f}_{\text{C-Sen}}, \boldsymbol{f}_{\text{C-SIC}}\}} G_{\text{tx,comm}} & \text{if both are feasible} \end{cases}$$

3. **Dual-Constraint Optimization** $(\mu \neq 0,\ \gamma \neq 0)$
   - → Optimal values $\mu_\star$ and $\gamma_\star$ must simultaneously satisfy sensing and SI constraints with strict equality

Restoring Constant-Modulus Constraints

- Starting with the Semi-Analytic Relaxed Solution

$$\boldsymbol{f}_\star = \sqrt{M_\mathsf{T}}\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \mu_\star\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \gamma_\star\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}\right]$$

- We find the CM-constrained solution

$$\boxed{\boldsymbol{f}_\star = \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\left\{\mathcal{D}\left[\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \mu_\star\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \gamma_\star\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}\right]\right\}}$$

The optimal multipliers are found **iteratively**

Our Approach

Refine the 3-step procedure to enforce CM constraints **at each iteration**

Modified 3-step Procedure

3-step Procedure

1. **Communication-Optimal Start** ($\mu = \gamma = 0$)
   - → Initialize with
   $$\boldsymbol{f} = \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}} \left\{ \mathcal{D} \left[\boldsymbol{H}_\mathsf{c}^H \boldsymbol{H}_\mathsf{c} \right] \right\}$$
   - → If feasible, stop.

2. **Single-Constraint Activation** ($\mu = 0$ or $\gamma = 0$)
   - → If only one constraint is violated, find the optimal multiplier μ or γ via Bisection Search

3. **Dual-Constraint Optimization** ($\mu \neq 0$, $\gamma \neq 0$)
   - → If both sensing and SI limits are active, refine the solution using a CM-adapted Newton-Raphson method.

Projected (Constant-Modulus) Bisection

- Let $\boldsymbol{X}$ be the objective and $\boldsymbol{Y}$ be the constraint
- Repeat
  1. $\omega \leftarrow (\omega_{\min} + \omega_{\max})/2$
  2. $\boxed{\boldsymbol{f} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}\left[(1-\omega)\boldsymbol{X} + \omega\boldsymbol{Y}\right]\}}$
  3. if $\boldsymbol{f}^H\boldsymbol{Y}\boldsymbol{f} > \beta$, then $\omega_{\max} \leftarrow \omega$
  4. else $\omega_{\min} \leftarrow \omega$
- Until $|\boldsymbol{f}^H\boldsymbol{Y}\boldsymbol{f} - \beta| \leq \epsilon$

Projected (Constant-Modulus) Newton-Raphson

- Repeat:

  1. $\boldsymbol{u} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}[(1-\boldsymbol{\alpha}^T\mathbf{1})\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \alpha_1\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \alpha_2\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}]\}$
  2. Set
  $$\boldsymbol{q}(\boldsymbol{\alpha}) \leftarrow \begin{bmatrix} \boldsymbol{u}^H\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t})\boldsymbol{u} - \tau^2 \\ \boldsymbol{u}^H\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}\boldsymbol{u} - \eta^2 \end{bmatrix}$$
  3. Compute the Jacobian matrix
  $$\boldsymbol{J}_{\boldsymbol{q}}(\boldsymbol{\alpha}) \leftarrow \begin{bmatrix} \frac{\partial}{\partial\boldsymbol{\alpha}[1]}q_1(\boldsymbol{\alpha}) & \frac{\partial}{\partial\boldsymbol{\alpha}[2]}q_1(\boldsymbol{\alpha}) \\ \frac{\partial}{\partial\boldsymbol{\alpha}[1]}q_2(\boldsymbol{\alpha}) & \frac{\partial}{\partial\boldsymbol{\alpha}[2]}q_2(\boldsymbol{\alpha}) \end{bmatrix}$$
  4. Update $\boldsymbol{\alpha} \leftarrow \boldsymbol{\alpha} - \boldsymbol{J}_{\boldsymbol{q}}^{-1}(\boldsymbol{\alpha})\boldsymbol{q}(\boldsymbol{\alpha})$
  5. $\boldsymbol{f} \leftarrow \mathcal{P}_{\mathbb{V}^{M_\mathsf{T}}}\{\mathcal{D}[(1-\boldsymbol{\alpha}^T\mathbf{1})\boldsymbol{H}_\mathsf{c}^H\boldsymbol{H}_\mathsf{c} + \alpha_1\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t}) - \alpha_2\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}]\}$

- Until $|\boldsymbol{f}^H\boldsymbol{a}_\mathsf{T}(\theta_\mathsf{t})\boldsymbol{a}_\mathsf{T}^H(\theta_\mathsf{t})\boldsymbol{f} - \tau^2| \leq \epsilon$ & $|\boldsymbol{f}^H\boldsymbol{H}_\mathsf{SI}^H\boldsymbol{H}_\mathsf{SI}\boldsymbol{f} - \eta^2| \leq \epsilon$

Numerical Example (I)

Simulation Setup:

- Communication Channel: narrowband clustered model
- SI Channel: Rice model (Near-Field LoS + Nearby Scatters NLoS)
- Arrays: $M_{\mathsf{T}} = 32$ (TX), $M_{\mathsf{S}} = 8$ (Radar), $M_{\mathsf{C}} = 16$ (Comm. RX)

Sensing Threshold:

$$\tau^2 \in [0, M_{\mathsf{T}}^2]$$

SI Threshold:

$$\eta^2 = \zeta M_{\mathsf{T}} s_{\max}(\boldsymbol{H}_{\mathsf{SI}}) \leq \zeta M_{\mathsf{T}} \|\boldsymbol{H}_{\mathsf{SI}}\|_{\mathrm{F}}^2 = \zeta M_{\mathsf{T}}^2 M_{\mathsf{S}} \text{ with } \zeta < 1$$

Benchmarks:

→ Upperbound: unconstrained without SI control (black)

→ SDR with CM constraints (red)

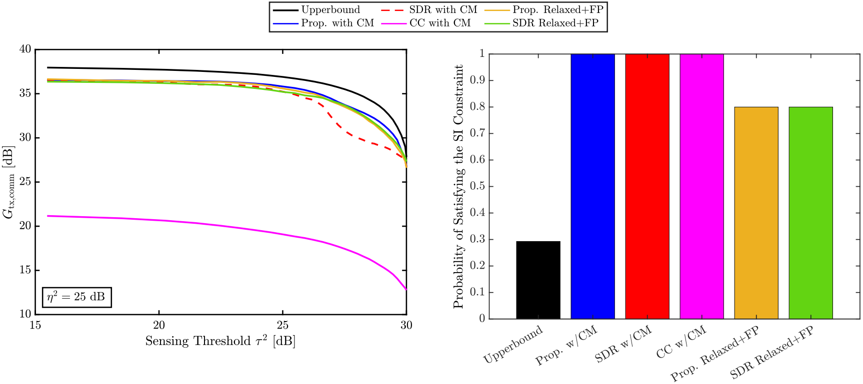
→ Convex Combination (CC) with CM constraints (magenta)

→ Relaxed with Final Projection (orange)

→ SDR Relaxed with Final Projection (green)

Numerical Example (II)

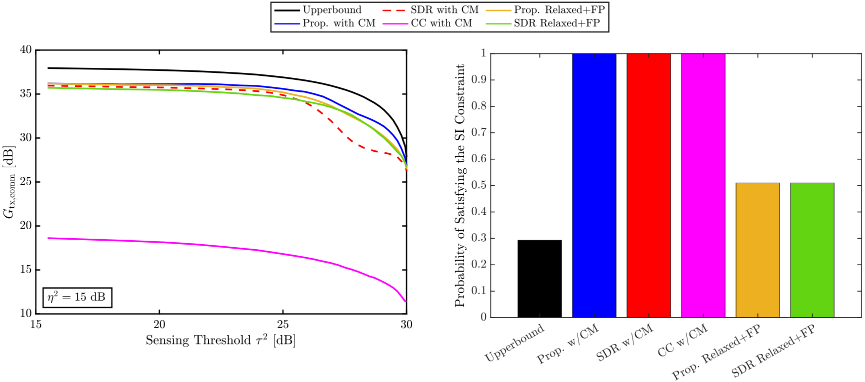
- Moderate SI Threshold (25 dB): $\eta^2 = \zeta M_{\mathsf{T}}^2 M_{\mathsf{S}}$ with $\zeta = \frac{1}{25}$



SDR with CM issues: complexity and tightness (60%)

Numerical Example (and III)

- Strict SI Threshold (15 dB): $\eta^2 = \zeta M_{\mathsf{T}}^2 M_{\mathsf{S}}$ with $\zeta = \frac{1}{250}$



SDR with CM issues: complexity and tightness (60%)

In Summary

1. We have addressed the design of multibeam analog beamformers for monostatic ISAC under SI
2. With per-antenna gain control (CM constraints relaxed), we have derived a semi-analytic solution in which only the Lagrange multipliers have to be numerically computed
3. For CM analog phased arrays, we have enforced CM constraints by projecting the relaxed solution onto the feasible set at each iteration
   - → Sensing and SI constraints simultaneously satisfied
   - → Performance closed to the upperbound

Backup: A Detail on Dual-Constraint Optimization

- Find $\mu, \gamma \in [0, \infty)$ so that both constraints are satisfied with equality
- Procedure:
  1. Reduce the search space

$$\alpha_1 = \frac{\mu}{1+\mu+\gamma} \in [0,1] \text{ and } \alpha_2 = \frac{\gamma}{1+\mu+\gamma} \in [0, 1-\alpha_1]$$

  2. Define $\boldsymbol{\alpha} \triangleq [\alpha_1, \alpha_2]^T$ and

$$\boldsymbol{u} \triangleq \sqrt{M_{\mathsf{T}}}\mathcal{D}[(1-\boldsymbol{\alpha}^T\mathbf{1})\boldsymbol{H}_{\mathsf{c}}^H\boldsymbol{H}_{\mathsf{c}} + \alpha_1\boldsymbol{a}_{\mathsf{T}}(\theta_{\mathsf{t}})\boldsymbol{a}_{\mathsf{T}}^H(\theta_{\mathsf{t}}) - \alpha_2\boldsymbol{H}_{\mathsf{SI}}^H\boldsymbol{H}_{\mathsf{SI}}]$$

  3. Find the roots of $\boldsymbol{q} : \mathbb{R}^2 \to \mathbb{R}^2$

$$\boldsymbol{q}(\boldsymbol{\alpha}) \triangleq \begin{bmatrix} q_1(\boldsymbol{\alpha}) \\ q_2(\boldsymbol{\alpha}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}^H\boldsymbol{a}_{\mathsf{T}}(\theta_{\mathsf{t}})\boldsymbol{a}_{\mathsf{T}}^H(\theta_{\mathsf{t}})\boldsymbol{u} - \tau^2 \\ \boldsymbol{u}^H\boldsymbol{H}_{\mathsf{SI}}^H\boldsymbol{H}_{\mathsf{SI}}\boldsymbol{u} - \eta^2 \end{bmatrix}$$

  via Newton-Raphson