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CSIT-Aware Spectral Precoding for Out-of-Band Power Reduction in OFDM

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Summary

- **What we propose:**
  - A CSIT-aware precoder that minimizes out-of-band power over band $\mathcal{B}$ under transmit-power and MI constraints.
  - Solved by a fixed-point algorithm combining reverse waterfilling and a semi-unitary front-end.
  - Spectral invariance under unitary rotations embeds CSIT without altering the transmit PSD.
  - Enables low-complexity ZF (SVD) and SIC (GMD) receivers.

System Overview

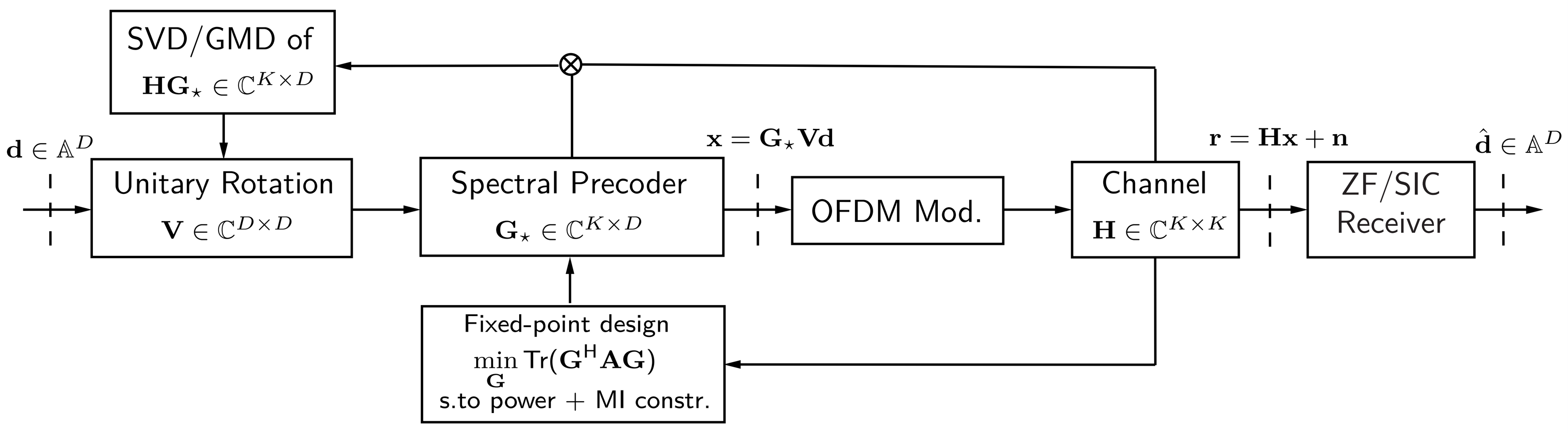

Fig.1 Proposed CSIT-Aware Spectral Precoding Chain.

- **Spectral precoding:**
  - Map $D \leq K$ symbols $\mathbf{d} \in \mathbb{A}^D$ to the active tones: $\mathbf{x} = \mathbf{Gd} \in \mathbb{C}^K$
  - $\mathbf{G} \in \mathbb{C}^{K\times D}$: redundancy $R = K - D$, rate $\eta = D/K \leq 1$.
- **PSD structure:**
  - Under $\mathsf{E}[\mathbf{d}_m\mathbf{d}_{m-\ell}^{\mathsf{H}}] = \delta[\ell]\mathbf{I}_D$, the PSD expression has the form [1]:

$$\boxed{P_s(f) \;\propto\; \mathbf{a}^{\mathsf{H}}(f)\,\mathbf{GG}^{\mathsf{H}}\,\mathbf{a}(f).}$$

  - $\mathbf{a}(f)$: stacked per-subcarrier spectral responses.
  - PSD depends on $\mathbf{G}$ through $\mathbf{GG}^{\mathsf{H}}$ (invariant to right-unitary rotations).
- **Channel:**
  - Under standard CP-OFDM, the post-FFT channel is diagonal:
$$\mathbf{H} = \mathrm{diag}\left\{H_c[k_1], \ldots, H_c[k_K]\right\} \in \mathbb{C}^{K\times K}.$$
  - $H_c[k]$ is the N-point DFT sample of the channel impulse response.
- **Received signal:**
  - The received vector (after CP removal and FFT):

$$\mathbf{r} = \mathbf{HGd} + \mathbf{n}, \quad \mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_K).$$

Optimization Framework

- **Three quantities at play:**
  - **Out-of-band radiation (OBR)** in the target band $\mathcal{B}$:
$$P_W = \mathrm{Tr}(\mathbf{G}^{\mathsf{H}}\mathbf{AG}), \quad \mathbf{A} \succeq \mathbf{0}$$
    where $\mathbf{A}$ captures leakage weighted by $W(f)$ in $\mathcal{B}$.
  - **Transmit power:** $\mathsf{E}[\|\mathbf{x}\|^2] = \mathrm{Tr}(\mathbf{G}^{\mathsf{H}}\mathbf{G})$.
  - **MI:** $\mathcal{I}(\mathbf{d};\mathbf{r}) = \log|\mathbf{I}_D + \mathbf{G}^{\mathsf{H}}\mathbf{CG}|, \quad \mathbf{C} \triangleq \frac{1}{\sigma^2}\mathbf{H}^{\mathsf{H}}\mathbf{H}$.
- **Design problem:**

$$\boxed{\min_{\mathbf{G}} \; \mathrm{Tr}(\mathbf{G}^{\mathsf{H}}\mathbf{AG}) \;\; \text{s.t.} \;\; \mathrm{Tr}(\mathbf{G}^{\mathsf{H}}\mathbf{G}) \leq P_{\max}, \; \log|\mathbf{I}_D + \mathbf{G}^{\mathsf{H}}\mathbf{CG}| \geq \mathcal{I}_{\min}.}$$

  - Non-convex.
  - Invariant to transformations $\mathbf{G} \to \mathbf{GV}$ for $\mathbf{V}$ unitary.
  - The MI constraint must be tight at the optimum.
  - Feasibility can be checked by waterfilling: $\mathcal{I}_{\min} \leq \mathcal{I}_0$, or equivalently by reverse waterfilling: $P_{\max} \geq P_0$.
- **First-order (KKT) condition:**
$$(\mathbf{A} + \mu\mathbf{I}_K)\,\mathbf{G} = \alpha\,\mathbf{CG}\,(\mathbf{I}_D + \mathbf{G}^{\mathsf{H}}\mathbf{CG})^{-1}, \quad \mu, \alpha \geq 0.$$
  - This condition motivates the proposed fixed-point iteration.
- **Solution insight:**
  - For each fixed $\mu \geq 0$, find $(\mathbf{G}_\mu, \alpha_\mu)$ satisfying KKT with the MI constraint tight.
  - Adapt μ to drive the transmit power to $P_{\max}$.
  - **Conjecture:** as $\mu\uparrow$, $\mathrm{Tr}(\mathbf{G}_\mu^{\mathsf{H}}\mathbf{G}_\mu)\downarrow$ and $\mathrm{Tr}(\mathbf{G}_\mu^{\mathsf{H}}\mathbf{AG}_\mu)\uparrow$.

Proposed Design

- **Fixed-point precoder algorithm**

Input: initial $\mathbf{G}_0$, $\mu_0 > 0$.
Repeat for $i = 1, 2, \ldots$ until convergence:
1. *Fixed-point update:*
$$\widetilde{\mathbf{G}}_i = (\mathbf{A} + \mu_{i-1}\mathbf{I}_K)^{-1}\mathbf{CG}_{i-1}(\mathbf{I}_D + \mathbf{G}_{i-1}^{\mathsf{H}}\mathbf{CG}_{i-1})^{-1}.$$
2. *MI scaling:* find $\alpha_i \geq 0$ such that $\log|\mathbf{I}_D + \alpha_i^2\widetilde{\mathbf{G}}_i^{\mathsf{H}}\mathbf{C}\widetilde{\mathbf{G}}_i| = \mathcal{I}_{\min}$.
3. *Precoder update:* $\mathbf{G}_i \leftarrow \alpha_i\,\widetilde{\mathbf{G}}_i$.
4. *Power-multiplier update:* $\mu_i \leftarrow \mu_{i-1} \cdot \dfrac{\mathrm{Tr}(\mathbf{G}_i^{\mathsf{H}}\mathbf{G}_i)}{P_{\max}}$.

Output: optimized precoder $\mathbf{G}_\star$ at convergence.

- **Unitary degree of freedom:**
  - Let $M = \mathrm{rank}(\mathbf{G}_\star) \leq D$. If $M < D$, retain the active modes of $\mathbf{G}_\star$.
  - For $\mathbf{V} \in \mathbb{C}^{M\times M}$ unitary, $\mathbf{G}_\star\mathbf{V}$ preserves PSD, MI, and power. $\mathbf{V}$ can be chosen to shape the effective channel $\mathbf{HG}_\star\mathbf{V}$ for ZF/SIC decoding.

Decoder-Aware Design

- **ZF receiver:**
  - Apply SVD: $\mathbf{HG}_\star = \mathbf{U}_h\mathbf{\Gamma}_h\mathbf{V}_h^{\mathsf{H}}, \quad \mathbf{\Gamma}_h = \mathrm{diag}\left\{\gamma_1, \ldots, \gamma_M\right\}$.
  - Transmit: $\mathbf{x} = \mathbf{G}_\star\mathbf{V}_h\mathbf{d} \quad \Rightarrow \quad \mathbf{r} = \mathbf{U}_h\mathbf{\Gamma}_h\mathbf{d} + \mathbf{n}$.
  - ZF detection: $\hat{\mathbf{d}} = \mathrm{DEC}\{\mathbf{\Gamma}_h^{-1}\mathbf{U}_h^{\mathsf{H}}\mathbf{r}\}, \quad \mathbf{w}_h \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{\Gamma}_h^{-2}\right)$.
- **SIC receiver:**
  - Apply GMD: $\mathbf{HG}_\star = \gamma\mathbf{Q}_h\mathbf{R}_h\mathbf{P}_h^{\mathsf{H}}, \quad \gamma = \left(\prod_{i=1}^M \gamma_i\right)^{1/M}$, where $\mathbf{R}_h$ is unit upper triangular and $\mathbf{Q}_h$, $\mathbf{P}_h$ (semi)-unitary.
  - Transmit: $\mathbf{x} = \mathbf{G}_\star\mathbf{P}_h\mathbf{d}$.
  - After projection: $\tilde{\mathbf{r}} \triangleq \gamma^{-1}\mathbf{Q}_h^{\mathsf{H}}\mathbf{r} = \mathbf{R}_h\mathbf{d} + \widetilde{\mathbf{w}}_h$, where $\widetilde{\mathbf{w}}_h \sim \mathcal{CN}\left(\mathbf{0}, \frac{\sigma^2}{\gamma^2}\mathbf{I}_M\right)$.
  - Backward SIC:
$$\hat{d}_M = \mathrm{DEC}\{\tilde{r}_M\}, \quad \hat{d}_\tau = \mathrm{DEC}\{\tilde{r}_\tau - \sum_{\nu>\tau}\rho_{\tau\nu}\hat{d}_\nu\}, \quad \tau = M-1, \ldots, 1.$$
- **Main benefit:** GMD-SIC equalizes the post-processing noise variance and avoids weak-mode ZF noise enhancement, with no PSD/OBR penalty.

Numerical Results

- **Simulation setup:**
  - **Waveform & channel:** CP-OFDM, rect. pulse, ideal LP DAC; $N = 128$, $l_g = 16$, $K = 65$ contiguous tones; flat $W(f)$ on $\mathcal{B}$; block-fading exp. PDP factor $\delta = 0.1$, $\sum_{k\in\mathcal{K}}|H_c[k]|^2 = K$, 16-QAM modulation.
  - **Constraints:** $P_{\max} = D$, $\mathcal{I}_{\min} = \zeta\,\mathcal{I}_0$, $\zeta \in (0,1)$; 10^3 ch. realizations.

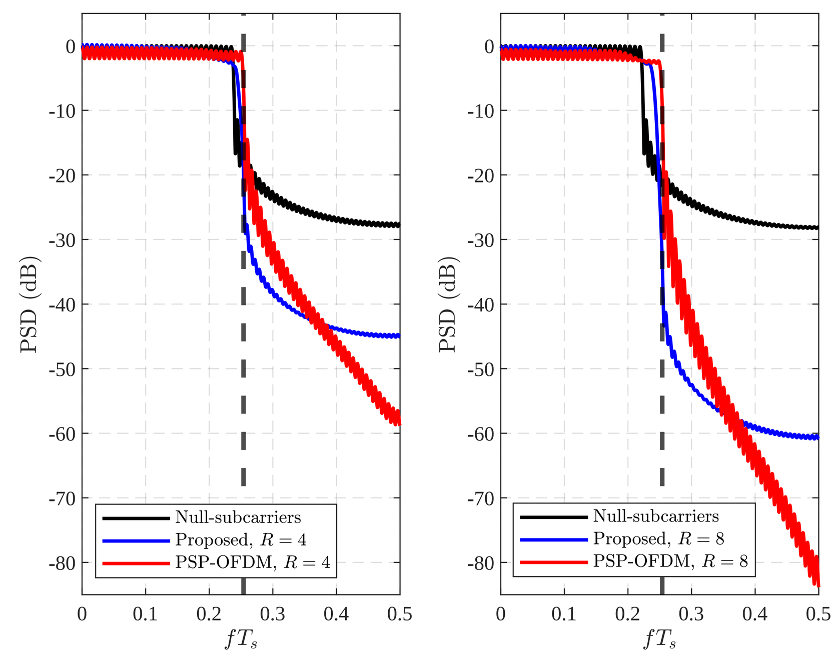

Fig. 2. PSDs of the prop. & PSP-OFDM [2] precoders with $R \in \{4, 8\}$, $\zeta = 0.99$.

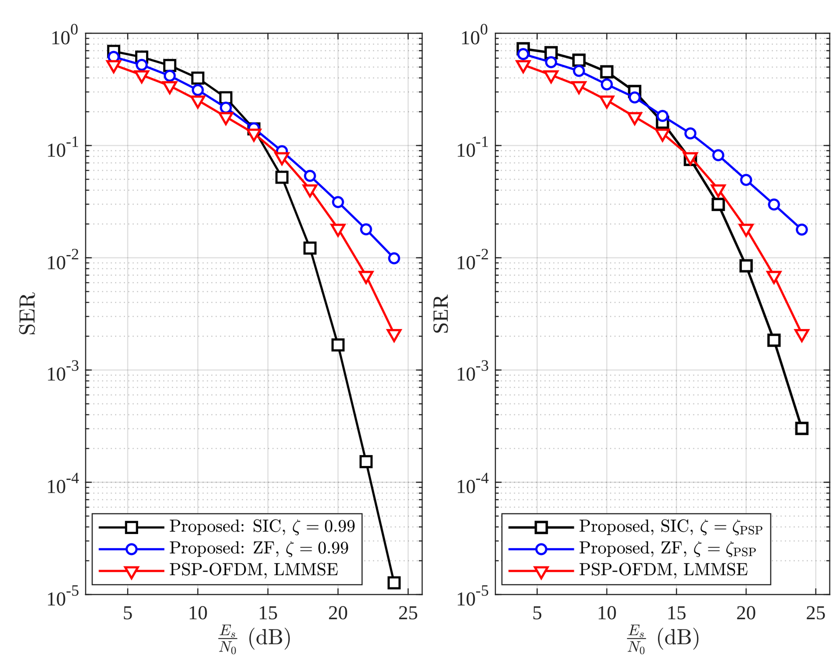

Fig. 3. SER of the prop. & PSP-OFDM [2] precoders, with $R = 6$, $\zeta \in \{0.99, \zeta_{\mathrm{PSP}}\}$.

ζ	0.95	0.96	0.97	0.98	0.99	0.995
OBR redu.	29.59	25.87	22.27	19.22	17.61	15.38

Table 1. Effect of MI: OBR reduction relative to plain OFDM (in dB).

References & Acknowledgment

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