PHY-MAC PERFORMANCE OF A MIMO NETWORK-ASSISTED MULTIPLE ACCESS SCHEME

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ABSTRACT

PHY-MAC analysis of the Feedback-Free Network-Assisted Diversity Multiple Access (FF-NDMA) scheme is tackled in this work, for both SISO and MIMO configurations with orthogonal space-time block codes. FF-NDMA was envisaged as a collision resolution technique for uncoordinated wireless ad-hoc networks. In the SISO case collision resolution is achieved by transmitting time diversity in different slots (repeating packets) while in the MIMO case both time and spatial diversity are exploited. In this paper it is shown that retransmission diversity may improve the collision channel, in terms of energy savings, range extension or bandwidth efficiency.

1. INTRODUCTION

The Network-Assisted Diversity Multiple Access (NDMA) protocol [1] provided an innovative signal processing-oriented solution for resolving collisions over the random access channel of cellular slotted systems. NDMA protocol does not discard colliding packets at the base station (BS) but combines them at the physical layer, thus requiring convenient BS-scheduled retransmissions to extract the information of each individual terminal. Under the assumption of perfect reception, NDMA protocol only requires as many uplink retransmission slots as colliding packets to resolve the collision. Thus, NDMA protocol dramatically enhances the throughput and delay performance of ALOHA-type random access schemes in spite of the overhead penalty needed to identify collisions multiplicity and the channel matrix. With such a method, the number of users, and so the receiver complexity. In [2-3] the authors concentrate on designing more efficient collision detection techniques. In [2] a collision detection method is addressed that overcomes the difficulty of orthogonal identification codes. Moreover, the proposed scheme is blind (B-NDMA) and introduces very modest overhead for collision detection. In [3] the performance of combining network and spatial diversity is investigated, assuming multiple antennas at the BS. In addition, a bandwidth preserving and blind collision detection method is proposed which does not require a tight retransmission process. A receiver architecture as well as retransmission strategies are proposed in [4] to detect collisions in dispersive channels. In [5] the stability analysis of these (B)NDMA protocols have been tackled.

The applicability of the NDMA protocol is not straightforward in uncoordinated ad-hoc networks (non cluster based), since scheduling information for retransmissions would also be exposed to collisions. A feedback free NDMA-based scheme is presented in [6], which is suited for uncoordinated broadcast transmission in ad-hoc networks.

None of these works introduce a joint PHY-MAC performance analysis and/or energy consumption issues. The investigation of these aspects implies the study of the receiver. These aspects are treated in this work for the FF-NDMA protocol [6] for the general case of MIMO structures with orthogonal space-time block codes. The decorrelating receiver is considered for simplicity of implementation and analysis. Due to the limited complexity possible at mobile terminals we concentrate on the analysis for terminals with two antennas at most. In the two transmit antennas case, the Alamouti transmission strategy is considered [8]. Other transmission strategies such as VBLAST are for further investigation.

In this paper it is shown that retransmission diversity may improve the efficiency in random access, either in terms of energy savings, range extension or bandwidth efficiency, in uncoordinated ad-hoc networks.

The FF-NDMA scheme is briefly described in the following section. The system model is settled in 3. The PHY-MAC analysis is performed in 4. The throughput is derived taking into account the PER obtained at the output of the receiver. In the following section performance results are discussed. Finally, section 6 is devoted to the conclusions.

2. FF-NDMA SCHEME

The multi-access scheme assumed is time slotted, so there is a need for synchronization. The broadcast channel is defined by a set of particular timeslots within allocated radio frames. A time slot in every frame or in a subset of all frames can be allocated for broadcasting. Figure 1 illustrates a possible radio frame assignment.
Throughout this section, Nand assumed Gaussian and white in time and space with variance through a linear receive simultaneously, i.e., nodes are half-duplex. Therefore, the channel transmission is required to be synchronized at contention period boundaries. Thus, synchronization at transmission is required as the contention period (CP) and the number of slots in a contention period R as the CP length (CPL), (see Figure 2). The optimum choice of the CPL was discussed in [6] for uncoordinated ad-hoc networks.

The following assumptions apply in the analysis:
1) Nodes have P neighbours and cannot transmit and receive simultaneously, i.e., nodes are half-duplex.
2) Users are restricted to start transmission of packets only at contention period boundaries. Thus, synchronization at contention period level is required. Synchronization at the symbol level is also assumed.
3) Packet generation is an independent process distributed geometrically with mean 1/αg, i.e., a node generates a packet in a CP with probability αg. Once a packet is generated its transmission is attempted in the next CP. The packet is transmitted in the R slots of the CP. When a node does not generate a packet, it listens to other nodes transmissions and demodulates them. So, the load offered in the broadcast channel to a node with P neighbours, defined as the average number of packets all the neighbours transmit per slot is G=αg/R (αg/R is the load per neighbour).
4) If a packet is received in error it is lost.
5) The channel coherence time is assumed much longer than the slot duration, but much shorter than the time between two consecutive slots allocated for the broadcast channel, and therefore, the channel may be considered constant on one slot and uncorrelated between slots (fast fading channel assumption). Channel is not known in transmission but known in reception.
6) The transmitted symbols are QPSK and the noise is assumed Gaussian and white in time and space with variance σ2.

3. SYSTEM MODEL

Throughout this section N and M denote respectively the number of receive and transmit antennas. When transmission is done through a linear STBC the transmitted signal matrix is given by:

$$X = \sum_{i=1}^{Q} \alpha_i A_i + j\beta \Pi_i$$

(1)

Q complex symbols (with real and imaginary part α and β, respectively, and unit power) are spread in time and space over T channel uses and the M transmit antennas through Q pairs of code matrices A and Π. These code matrices can be chosen real-valued for OSTBC [7] (A, Π ∈ ℝM×1).

If the channel is flat fading, signal detection may be performed block by block. In a single user channel, the received signal on any block may be written in vector form as [9], page 98

$$y = A/\sqrt{L}/2M \bar{H}x + w$$

(2)

where L stands for the propagation losses, y and w ∈ ℝ2NT×1 gather the real and imaginary parts of the signal and noise samples received at all the N sensors during the T channel uses, x=[α² β²]T contains the 2Q real symbols of the block and \( \bar{H} \in \mathbb{R}^{2NT×2NT} \) is an equivalent channel matrix resulting from combination of ST coding and the Rayleigh flat-fading channel matrix, \( \bar{H} \in \mathbb{C}^{N×N} \), which has zero mean complex Gaussian components. Note that the transmitted energy per symbol is fixed to \( E_s = \bar{A}^T \), regardless the number of antennas.

If any OSTBC is used along with the combination of retransmissions the received signal in a contention period of length R slots, where K users are present, may be arranged in vector form as follows

$$y = SAx + w$$

(3)

where x=[x₁ ... x_K]T ∈ ℝQK×1 contains the blocks of real symbols transmitted by the K users, y=[y₁ ... y_R]T and w=[w₁ ... w_R]T ∈ ℝ2NT×1 contain the real and imaginary parts of the received signal and noise samples in the R slots of the CP and \( \Lambda = \text{diag}(\{A/\sqrt{L_e}\}_{1Q} \ldots \{A/\sqrt{L_e}\}_{1Q})/\sqrt{4M\bar{R}} \) represents the pathloss between the k-th transmitter and the receiver. Finally,

$$S = \sqrt{2} \begin{bmatrix} \bar{H}_{1r} & \cdots & \bar{H}_{kr} & \cdots & \bar{H}_{1e} & \cdots & \bar{H}_{ke} \end{bmatrix} \in \mathbb{R}^{2NT×2KNT}$$

(4)

where \( \bar{H}_{kr} \) is the equivalent channel matrix of user k during the r-th time slot (refer to (10)). Note that the transmitted energy per symbol is fixed with and without retransmissions, so as comparisons in the sequel are fair in terms of energy consumption. Note that the system model in (3) is also valid for the SISO case by setting up N=M=R=1.

4. PHYSICAL-MAC ANALYSIS

4.1. Bit Error Rate (BER)

By using the decorrelating receiver, the MAI is vanished and the bit-error rate is invariant to the amplitudes of the interfering signals. Then, the bit-error rate of user k at the output of the decorrelator \( BER_k \) may be expressed as

$$BER_k = Q\left(\sqrt{E_s/N_e}\right) = Q\left(\frac{1}{2RM/\sigma^2} A_k^T L_k x_k \right)$$

(5)

where \( x_k \) is a random variable with chi-square distribution with \( D=2(RNM-QK+Q) \) degrees of freedom when the Alamouti strategy is used and in general for any OSTBC with \( M<T \). Note that this expression also holds for the SISO case (refer to the
Note that while $R$ is a value to be fixed for network operation, the value of $K$ is in fact random in each contention period and depends on the offered load. So, the BER in a CP depends not only on the power transmitted by user $k$ but also on the actual number of users transmitting $K$, although it is independent of the transmitted power by the $K$-1 interfering users.

### 4.2. Packet Error Rate (PER)

For the PER analysis, it will be assumed that a received packet is in error whenever the BER is above a certain threshold value $v$. If $v$ is low enough this gives an approximated upper bound for the packet error and allows evaluating the performance independently of the underlying channel code. According to that and (5) we may set

$$PER_k = \Pr\left(\sqrt{Z_k} \leq Q^{-1}(v)\sqrt{\frac{2RM}{A_k^\theta L_k}}\right) = 1 - F_\chi\left(\sqrt{R_k}\right)$$

where $F_\chi$ is the cumulative function of the chi distribution

$$F_\chi(x) = e^{-\frac{x^2}{2}} \sum_{l=0}^{\infty} \frac{\left(\frac{x^2}{2}\right)^l}{l!}, \quad l = \frac{D}{2} - 1$$

and

$$x = Q^{-1}(v)\sqrt{\frac{2M}{A_k^\theta L_k}}$$

### 4.3. Throughput

The throughput, $S_{R,P}$, of received packets for a node with $P$ neighbours (each one repeating a packet in $R$ slots) is defined as the average number of correctly received packets per slot

$$S_{R,P} = \frac{1}{R}(1 - \alpha_e) \sum_{k=1}^{\text{max}(h_P)} k p_e(k) F_{\text{PSR}}(1 - \alpha_k, \text{CP})$$

where $F_{\text{PSR}}(1 - \alpha_k, \text{CP})$ is the Packet Success Rate ($PSR=1$-PER) as defined in (7) a function of the number of transmitting users, $k$, in each contention period $R$, the term $1 - \alpha_k$ is due to the half-duplex operation of nodes, and

$$p_e(k) = \binom{P}{k} \alpha_k^k (1 - \alpha_k)^{P-k}$$

is the probability that $k$ over $P$ nodes transmit a packet in a CP. Note that in (8) packets are unsuccessfully received if more than $R$ transmissions occur in a CP of length $R$ (this is a worst case approximation as it is not considering capture effects). $R$ is equal to $R$ in the SISO case as in [6] and $2R$ in the MIMO $2 \times 2$ case (to keep full column rank of $S$). Note that (8) is modified from equation (3) in [6], by weighting each summation term with its $PSR$, which is a function of the $E_s/N_0$. Equation (8) is a lower bound in performance if the average $E_s/N_0$ of the neighbor in the worst conditions is considered, but it is an exact expression if assumed that every neighbor is received with the same average $E_s/N_0$.

### 4.4. Energy Efficiency

The quotient between the throughput and the offered load gives a measure of the energy efficiency $E_{R,P}$ of the protocol:

$$E_{R,P} = \frac{S_{R,P}}{G_{R,P}}$$

$E_{R,P}$ is a measure of the energy percentage wasted to demodulate packets, i.e., unnecessarily consumed due to an excess of resolvable users in a CP, due to a bad channel state or due to unavailability situations.

### 5. PHYSICAL-MAC PERFORMANCE

The performance of the FF-NDMA scheme was clearly overestimated in medium-to-low $E_s/N_0$ conditions in [6], since the channel was assumed to be error free. The throughput values obtained there correspond to high $E_s/N_0$ levels and are only valid for SISO schemes. In the MIMO $2 \times 2$ case implementing the Alamouti transmission strategy, extra degrees of freedom are available to resolve more users per CP (up to $2R$), then the throughput is expected to increase and its maximum should tend to double. Regarding the optimum length of the contention period (CPL), it is expected to be reduced. This occurs because twice the users than in the SISO case may be solved in a CP. Lower $E_s/N_0$ requirements than in the SISO case are also expected (which is traded in power consumption savings or better connectivity) for a fixed achievable throughput. This can be easily proved by evaluating the degrees of freedom in (5) for the SISO case ($D=2$) and for the MIMO case ($D=4$).

![Figure 3. Throughput versus offered load: SISO (solid lines) and Alamouti-MIMO 2x2 (dashed lines) configurations. R and 2R users respectively are resolvable in a CP. Plots are for P=12 neighbours. High E_s/N_0 is considered.](image-url)
case, since $N_M > 1$. On the MAC layer side, an increase of nodes activity implies a larger rate of unavailability situations, i.e., nodes pass most of their time transmitting and not hearing their neighbours. This MAC effect is less critical for the studied MIMO schemes since the unavailability collisions are compensated by a higher efficiency in the number of resolvable packets per CP.

**Figure 4.** Energy efficiency versus received $E_r/N_0$(dB) for $G=0.2$ ($P=7$): a) SISO, b) MIMO 2x2 (2R resolvable users in a CP)

Those effects are illustrated through Figure 4 and Figure 5 where the energy efficiency of a) SISO and b) MIMO 2x2 (up to $2R$ resolvable users) configurations are plotted versus the received $E_r/N_0$(dB) for a value of $\nu$ fixed to $10^{-3}$ (refer to equation (6)). Two different loads are depicted, a low load $G=0.2$ in Figure 4 and a medium load $G=0.6$ in Figure 5. Throughput values may be obtained directly from these plots by multiplying by the fixed value of $G$ (refer to (9)). In all cases $P=7$.

In the SISO case, for a given load the maximum achievable throughput for $R=1$ may always be obtained for some $R>1$ with lower required energy per symbol. It means that energy savings or better connectivity may be achieved when using $R>1$. For instance, in Figure 4 a) it may be observed that a throughput of $0.16$ (0.8 energy efficiency) may be achieved with a reduction in $E_r/N_0$ of 10 to 13 dB when $R$ are taken as 2 to 7, respectively, as compared to the case of $R=1$. This means that for energy limited systems, we can use $R>1$ to save battery at the expense of throughput (note the low values of the load).

Note also that at low load the set of values of $R>1$ providing energy savings is higher than at high load ($R$ ranges from 2 to 7 in Figure 4a) while from 2 to 5 in Figure 5a)). This is due to the increase of unavailability collisions with $R$ when the load increases. For non energy-limited systems (able to operate in the high $E_r/N_0$ range) a higher throughput may be achieved than in the case $R=1$ when setting $R>1$ as it was shown in [6].

It may be observed in Figure 4 and Figure 5 b) that in the MIMO 2x2 case, the higher throughput may be achieved with $R=1$. However note that the optimum value of $R (R_o)$ is related to the number of neighbours of terminals $P$ (refer to (8)). It was shown in [6] for the SISO case that $R_o$ increases as $P$ increases but with lower slope than 1. That is also true for MIMO configurations, but when considering up to $2R$ resolvable users, $R_o$ increases at a lower rate. For $P = 7(12)$, $R_o$ is $3(4)$ for SISO while 1(2) for the MIMO scheme considered in b) as deduced from Figure 3-Figure 5. So, as $P$ increases (high dense networks) gains from retransmission combining, as reported for SISO schemes, in terms of energy consumption may be also achieved for MIMO ones. However, note the reduction of the required received energy per symbol when using the MIMO 2x2 configuration in b) occurs for any value of $R$. As a consequence, energy savings in the MIMO case are lowered respect to the SISO case when using $R=1$.

6. CONCLUSIONS

In this paper, we have analysed the PHY-MAC performance of the FF-NDMA collision resolution scheme. The analysis has been carried out for the decorrelator receiver in frequency flat
channels. We have shown that retransmission diversity (R>1) may improve the efficiency of random access, either in terms of energy, connectivity or bandwidth efficiency, in uncoordinated ad-hoc networks. Retransmission combining gains are more important for the SISO case especially in low dense networks. In the MIMO case additional gains are obtained in terms of required $E_b/N_0$ and throughput.

Collision resolution when other transmission strategies are used when MIMO structures are available is a work in currently progress. Analysis of achievable connectivity depending of node density is another future research topic.

APPENDIX

The elements of $\tilde{H}$ in (2) may be expressed as

$$\begin{bmatrix}
\mathbf{h}_{j,q}^T \\
\mathbf{h}_{i,q}^T \\
\mathbf{h}_{j,q}^T
\end{bmatrix}_n = \begin{cases}
1 & i = N(t-1) + n, j = q \\
2 & i = N(t-1) + n, j = q + 1 \\
0 & \text{else}
\end{cases}$$

where $\mathbf{h}_{j,q}$ and $\mathbf{h}_{i,q}$ are the $n$-th row of the real and imaginary part of the channel matrix $\mathbf{H}$ respectively, and $\mathbf{a}_n$ and $\mathbf{b}_n$ are the $t$-th columns of the $q$-th code matrixes pairs, $\Lambda_q$ and $\Pi_q$ respectively. It follows that the elements of $\mathbf{H}$ are real Gaussian, and their variance is either $\frac{1}{2}||\mathbf{a}_n||^2$ or $\frac{1}{2}||\mathbf{b}_n||^2$. For the OSTBC codes where $T=M$, the set of column vectors of $\Lambda_q$ and $\Pi_q$ are orthonormal, so, the variance of the elements of $\mathbf{H}$ is $\frac{1}{2}$ and the elements of a column are uncorrelated.

So, for the Alamouti code (in general for OSTBC with $T=M$) and the SISO case each of the columns of $\mathbf{S}$ in (4) is a multivariate real Gaussian r.v. with covariance matrix $\mathbf{I}_{2M}$.

In addition for OSTBC and the SISO case the columns of $\mathbf{H}$ in equation (2) are orthogonal. Due to that, all the columns of $\mathbf{S}$, associated to the retransmissions of a given user are orthogonal as well.

$$x_n = \sum_{m=1}^{2M} (\mathbf{S}^H \mathbf{S})_{mn} \mathbf{y}_m$$

may be written as (10), p. 196:

$$X_s = \mathbf{s}_n^T \mathbf{P} \mathbf{s}_s$$

where $\mathbf{s}_n$ is the $n$-th column of $\mathbf{S}$ and

$$\mathbf{P} = \mathbf{S}_s (\mathbf{S}_s^H \mathbf{S}_s)^{-1} \mathbf{S}_s^T$$

where $\mathbf{S}_s$, $\mathbf{s}_s$ has been removed from $\mathbf{S}$.

Let $\mathbf{S}_s$ be decomposed as $\mathbf{S}_s = [\mathbf{A} \ \mathbf{B}]$, where $\mathbf{A}$ contains the $2Q-1$ columns orthogonal to $\mathbf{s}_s$ and $\mathbf{B}$ the columns of other users. If $\mathbf{U}_s$ spans the $2Q-1$ dimensional space orthogonal to $\mathbf{s}_s$, then

$$\mathbf{P} = \mathbf{S}_s (\mathbf{S}_s^H \mathbf{S}_s)^{-1} \mathbf{S}_s^T = \mathbf{U}_s \mathbf{U}_s^H + \mathbf{U}_s \mathbf{U}_s^H$$

If $\mathbf{P}_s^\perp$ denotes the projection matrix orthogonal to the space spanned by $\mathbf{U}_s$ it follows that

$$X_s = \mathbf{s}_s^H \mathbf{P}_s^\perp \mathbf{s}_s = \mathbf{s}_s^H \mathbf{P}_s^\perp \mathbf{s}_s$$

by combining (11) and (13). This projection matrix may be written as in terms of its eigendecomposition, following that

$$X_s = \sum_{i=1}^{2MN-2QRQ(N-1)} |s_i^s v_i|^2$$

where $V=\{v_1, ..., v_{2MN-2QRQ(N-1)}\}$ is an orthonormal basis orthogonal to the space spanned by the columns of $\mathbf{U}_s$. The vectors in $V$ are independent of $\mathbf{s}_s$. Therefore, the products $s_i^s v_i$ are i.i.d random variables, which conditioned on $\mathbf{v}_i$ are zero-mean real Gaussian with unit variance.

REFERENCES