In this paper, a simple method for generating correlated Rayleigh-fading envelopes with any desired correlation factor is proposed. In addition, based on a process of apparition and vanishing of same delay correlated rays, a time-varying impulse response model of the indoor radio channel is developed with the aid of the proposed method. This proposal provides a straightforward tool for simulating the variant behavior of wireless channels.

Keywords – correlation factor; multipath propagation; Rayleigh distribution; time variant channel

I. INTRODUCTION

For simplicity, in the simulation of wireless communications systems the small-scale parameters of the channel are commonly modeled as uncorrelated random processes, but in general, there is some correlation degree which exerts strong influence on the performance of wideband, diversity and combining systems. In order to obtain realistic results, simulation tools for the analysis of wireless systems must consider certain correlation degree such that the statistical dependence of the channel’s parameters will be properly represented.

In this paper, we deal with the problem of generating small-scale correlated amplitudes (envelopes) for wireless channel simulation purposes. Because Rayleigh distribution is frequently employed for modeling the channel’s envelopes, we propose a straightforward method for generating correlated Rayleigh envelopes with any desired correlation factor. The idea underlying this proposal is the recursive generation of correlated complex Gaussian random samples such that their magnitudes will be Rayleigh distributed and correlated with one another by a specific factor. With the aid of this method, a frequency-selective time-varying impulse response model of the indoor radio propagation channel is developed for multicarrier CDMA (MC-CDMA) applications at 2.4 GHz.

Such impulse response model is the result of combining a realistic discrete-time impulse response model of the indoor radio propagation channel with a process of apparition and vanishing of same delay correlated multipath components.

The sequel of this paper is as follows. In section II a brief description of some important concepts of the indoor radio channel are given. In section III the proposal is explained when narrowband channels are considered. In section IV an application of the proposal is given for wideband channels. In section V the indoor radio channel’s time-varying impulse response model is described highlighting the applications of the proposal and the apparition and vanishing process. Finally, the conclusions on the results obtained in this paper are given.

II. INDOOR RADIO PROPAGATION CHANNEL

Because the indoor radio propagation channel can be viewed as a linear time-varying filter, it can be represented by means of its impulse response as [1]:

\[ h(t, \tau) = \sum_{n=1}^{N_{\text{mp}}(t)} \alpha_n(t, \tau) \delta(t - \tau_n(t)) \]  

where \( h(t, \tau) \) is the channel’s response at time \( t \) to an impulse applied at time \( t - \tau \). \( N_{\text{mp}}(t) \) is the number of multipath components. \( \alpha_n(t, \tau) \) and \( \tau_n(t) \) are the complex attenuation factor and propagation delay of the \( n \)th multipath component respectively.

Indoor radio channels can be classified as narrowband channels when the bandwidth of the transmitted signal \( B_t \) is less or equal than the channel’s coherence bandwidth \( B_c \) and as wideband channels when \( B_t \) is bigger than \( B_c \). Although narrowband channels can be accurately characterized by (1), in this paper narrowband channels are represented as:

\[ h(t) = \alpha(t) \delta(t - \tau) \]  

Because of changes in the propagation environment, the indoor radio channel is time and space variant. Spatial variations are produced by changes in the spatial position of the transmitter and receiver within a static propagation environment. Temporal variations are caused by the motion of people and equipment (scatterers) when the transmitter and receiver are fixed. The variability of the channel is closely related to the correlation potential of its parameters either in time and space; hence the importance of having a simple method to provide channel simulators with correlation features.
III. GENERATION OF CORRELATED ENVELOPES

A. Summary of previous proposals

A useful method for generating two equal power correlated Rayleigh envelopes was given in [2]. This method employs a coloring matrix for getting a vector $\mathbf{x}$ of two correlated complex Gaussian elements in such a way that their envelopes are cross-correlated by a specific value. The coloring matrix is a lower triangular matrix obtained by performing the Cholesky decomposition on the required correlation matrix of $\mathbf{x}$.

In [3], the coloring matrix proposed in [2] was modified to represent situations where the correlation factor of the Gaussian samples is complex. In [4], an extension of [2] when $\mathbf{x}$ has $N$ elements was given considering a real correlation matrix $\mathbf{R}_x$ of $\mathbf{x}$; a similar method but for a complex $\mathbf{R}_x$ was given in [5]. Recently, a recursive method for generating a vector $\mathbf{x}$ of $N$ complex Gaussian elements with correlated Rayleigh envelopes was proposed in [6] considering a real $\mathbf{R}_x$. In this section, we extend the proposal of [6] to cover the case of complex $\mathbf{R}_x$.

The main advantage of the method proposed here, is the replacement of the $N \times N$ coloring matrix calculation process of [4] and [5] by a simple recursive algorithm which considerably reduces the computational costs.

B. Problem and theoretical description of the proposal

Suppose that we want to generate a vector $\mathbf{x} = (x_0, x_1, \ldots, x_N, y_0, y_1, \ldots, y_N)^T$ of $N$ complex Gaussian elements to represent time samples of the time-varying narrowband channel. Moreover, we want the adjacent elements of $\mathbf{x}$ to be cross-correlated such that their envelopes (given as $r_m = |x_m|$, where $| \cdot |$ denotes the complex absolute value) will be correlated by a given value.

Under those requirements, the correlation factor of $x_{k-1}$ and $x_k$ ($\gamma_{k-1,k}$) will depend on that of $r_{k-1}$ and $r_k$ ($\rho_{k-1,k}$), thereby, it is necessary to know the relationship between $\gamma_{k-1,k}$ and $\rho_{k-1,k}$, which according to [2] and [4] can be found with the aid of the following equation:

$$
\rho_{m,k} = \frac{\left(1 + \gamma_{m,k}\right)E(\sqrt{\gamma_{m,k}})}{2 - \frac{\pi}{2}} - \frac{\pi}{2}
$$

where $m, k = 0, 1, 2, \ldots, N-1$ and $E(.)$ denotes the complete elliptic integral of the second kind. Considering (3), a root-finding algorithm was employed in [2] to compute $\gamma_{m,k}$ from a given value of $\rho_{m,k}$, while in [4] an empirical method was suggested to estimate $\rho_{m,k}$ from a given value of $\gamma_{m,k}$. In [3], two explicit closed-form approximations were provided when the relationship between $\rho_{m,k}$ and $\gamma_{m,k}$ is given by the hypergeometric function. It must be noted that for the processes given in [2-4] it is enough to know the complex absolute value of $\gamma_{m,k}$.

For the case that we have posed, once the desired value of $\rho_{k-1,k}$ has been chosen for $k = 1, 2, \ldots, N-1$, the corresponding value of $\gamma_{k-1,k}$ can be estimated with the aid of (3) and the problem simply turns into generate $N$ correlated complex Gaussian samples where the correlation factor between $x_{k-1}$ and $x_k$ is given by the computed value of $\gamma_{k-1,k}$.

For this reason, we propose a method where $x_k$ can be obtained by means of a direct relation with $x_{k-1}$ taking into account that the correlation factor of $x_{k-1}$ and $x_k$ must be equal to $\gamma_{k-1,k}$. We consider that $x_{k-1}$ and $x_k$ have zero mean and equal variance $\sigma$ and we suppose that a vector $\mathbf{v} = (v_0, v_1, \ldots, v_N, y_0, y_1, \ldots, y_N)^T$ of uncorrelated zero-mean complex Gaussian elements with variance $\sigma$ is available. For the sake of clarity in the explanation of the process derivation, we will analyze first the case of two complex Gaussian samples $x_0$ and $x_1$.

Having into account the considerations mentioned above, the correlation matrix $\mathbf{R}_x$ of $\mathbf{x} = (x_0, x_1)^T$ can be expressed as:

$$
\mathbf{R}_x = \begin{bmatrix}
1 & \gamma_{0,1} \
\gamma_{0,1}^* & 1
\end{bmatrix}
$$

where $(\cdot)^*$ denotes the complex conjugate. Performing the Cholesky decomposition on (4) to obtain a lower triangular matrix $\mathbf{L}$ such that $\mathbf{L}^\mathbf{H} = \mathbf{R}_x$, it can be easily found that:

$$
\mathbf{L} = \begin{bmatrix}
1 & 0 \\
\gamma_{0,1} & \sqrt{1 - |\gamma_{0,1}|^2}
\end{bmatrix}
$$

Since in agreement to [2] $\mathbf{x} = \mathbf{L} \mathbf{v}$, where $\mathbf{v} = (v_0, v_1)^T$; then, we have that $x_0$ and $x_1$ are given as:

$$
x_0 = v_0
$$

$$
x_1 = \gamma_{0,1} v_0 + v_1 \sqrt{1 - |\gamma_{0,1}|^2}
$$

As can be seen in (6) and (7), $x_0$ directly takes the value of $v_0$ and $x_1$ is easily obtained by means of a combination of $v_0$ and $v_1$. If both equations are properly handled, we can extend this process to obtain $N$ correlated complex Gaussian samples. This is because if we have previously generated $x_{k-1}$ we can get $x_k$ defining $\mathbf{v} = (x_{k-1}, v_k)^T$ and repeating the process for consecutive values of $k$. In fact, since every time this process is performed for a new $\gamma_{k-1,k}$ and given that the values of the previously generated variables don’t change, the whole algorithm can be simplified by rearranging (7) as:

$$
x_k = \gamma_{k-1,k} x_{k-1} + v_k \sqrt{1 - |\gamma_{k-1,k}|^2} \quad \text{for } k = 1, 2, \ldots, N-1
$$

With the aim of providing our algorithm with a realistic profile and knowing that the elements of $\mathbf{x}$ can be expressed as:

$$
x_k = a_k + j b_k \quad : k = 0, 1, \ldots, N-1
$$

where $a_k$ and $b_k$ are two independent zero-mean Gaussian random variables with same variance $\sigma^2/2$, and in order for the envelopes of $\mathbf{x}$ to be equal power correlated Rayleigh random variables, the following relationships must be observed [2,5]:

for $k = 0, 1, 2, \ldots, N-1$:

$$
E[a_k^2] = E[b_k^2] = \sigma^2 / 2
$$

$$
E[a_k b_k] = 0
$$

for $k = 1, 2, \ldots, N-1$:

$$
E[a_k a_k] = E[b_k b_k] = \varphi_{k-1,k}
$$

$$
E[a_k b_k] = -E[b_k a_k] = \phi_{k-1,k}
$$
Choosing $\varphi_{k-1,k} = K\gamma_{k-1,k}$ ($K$ is a constant) allows the simulation of different transmission conditions [3]; thereby, $\gamma_{k-1,k}$ may be expressed as:

$$\gamma_{k-1,k} = \frac{2\gamma_{k-1,k}(1 - jK)}{\sigma^2}; \quad k = 1, 2, \ldots, N-1$$  (11)

Substituting (11) in (8) and expressing $x_k$ in terms of the complex absolute value of $\gamma_{k-1,k}$, we finally have that:

$$x_k = x_{k-1}\left[\frac{(1 + jK)}{\sigma^2} + j\sqrt{1 - \gamma_{k-1,k}^2}\right]$$  (12)

for $k = 1, 2, \ldots, N-1$. In [3], the value of $K$ was chosen for simulating frequency correlation making $K = -\Delta\omega\sigma$, where $\Delta\omega$ is the frequency separation of the correlated Gaussian samples and $\sigma$ is a measure of the channel’s delay spread. When there is no frequency separation $K = 0$.

C. Summary of the algorithm and simulation examples

The algorithm for getting $x$ can be summarized as follows:
1) Given the required value of $\rho_{k-1,k}$ find the corresponding value of $\gamma_{k-1,k}$ for $k = 1, 2, \ldots, N-1$. References [2] and [4] provide tables and two equations are given in [3] to cover this task.
2) Generate $v$ having into account the desired value of $\sigma^2$.
3) Finally, compute $x$ with the aid of (6) and (12) choosing the value of $K$ as needed\footnote{For the simulation of narrowband indoor radio channels $K \approx 0$ when the delay spread and/or the Doopler shift can be neglected.}. In this way, the envelopes of $x$ will be correlated according to the raised case.

In fig. 1 the Rayleigh probability density function (pdf) is compared with that of 50,000 envelopes obtained with our algorithm for $\rho_{k-1,k} = 0.95$, $K = 0$ and $\sigma^2 = 2$. As can be seen, the empirical pdf closely approximates the theoretical one.

Since for real channels the phase changes markedly during deep amplitude fades [3], this behavior must be reproduced by any valid channel simulation technique. In fig. 2 the envelopes and phases of 1,000 correlated Gaussian samples generated using our algorithm are depicted. As must be, drastic phase changes can be observed for deep envelope fades.

D. Correlation properties of the proposed algorithm

The resulting correlation matrix of $x$ when our algorithm is employed can be written as:

$$R_{xx} = \begin{bmatrix}
1 & \gamma_{0,1} & \gamma_{0,2} & \cdots & \gamma_{0,N-1} \\
\gamma_{0,1} & 1 & \gamma_{1,2} & \cdots & \gamma_{1,N-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\gamma_{N-1,0} & \gamma_{N-1,1} & \cdots & 1
\end{bmatrix}$$  (12)

where the correlation factor between $x_k$ and $x_m$ for $k, m = 0, 1, 2, \ldots, N-1$ and $k < m$, is given by:

$$\gamma_{k,m} = \prod_{n=k}^{m-1} \gamma_{k,n+1}$$  (13)

Hence, the proposed algorithm is restricted to applications where the correlation matrix of the Gaussian samples can be expressed in the form of (12) with elements given by (13). Fortunately, most time and frequency correlation matrices fulfill this requisite (the algorithm can be easily extended to simulate frequency correlation making $K \neq 0$ [3]). For space correlation the algorithm can be employed only for linear antenna array considerations. Despite of its restrictions, the simplicity and computational saving makes our proposal a convenient option.

IV. WIDEBAND CHANNELS APPLICATION

The wideband channel model given by (1) can be seen as the sum of $N_{mp}$ narrowband channels with different propagation delays (rays). In this way, similar delay rays will exhibit some correlation value which decreases as delay increases, thereby, far enough rays will be independents.

Because the propagation delay can take an infinite number of values within the interval defined from the line-of-sight (LoS) delay to the channel’s maximum excess delay, for computer simulators implementations it is convenient to limit it to a finite number. The discrete impulse response model of the channel [1], where the time and delay axis are divided into small intervals called bins, copes that problem. In this model, a single ray represents all rays contained in the same time or delay bin.
In order to provide this model with time (and delay) correlation properties, the recursive method explained in section III may be employed. In this way, $N_m$ time correlated narrowband channels are generated, each one representing a particular delay bin. The time correlation factor depends on the length of the time bins, hence, for long bins the correlation factor will be low whereas for short bins it will be high [1]. The same considerations apply for delay correlation, although in practice the correlation between adjacent delay rays is low enough to consider them as uncorrelated (correlation fewer than 0.25 for 5ns delay bins [1,8]).

With the aim of explaining how to simulate wideband time-variant channels, we provide an example for MC-CDMA applications in the unlicensed ISM band.

We considered for the model the parameters of the MC-CDMA/vsf system described in [9]. There, the whole 83 MHz bandwidth is divided in two channels of 40 MHz. The MC-CDMA/vsf signal is sampled every 25 ns yielding to delay bins of 25 ns and considering slow-fading propagation time bins will be 4 μs length, which is given by the MC-CDMA/vsf symbol’s time duration. Because delay bins have a length of 25 ns, we supposed delay-uncorrelated rays while each ray’s adjacent time samples were supposed to exhibit a correlation factor of 0.95; that value was chosen to highlight the effect of employing the algorithm of section III, for realistic simulations this value must be lower. The channel was supposed to have an exponential decay power delay profile and it was considered that no rays beyond 800 ns were received (maximum excess delay allowed). The interpath arrival was typified by an exponential pdf. For the simulation, 50 MC-CDMA/vsf symbols and a delay observation interval of 3.2 μs were chosen. Since rays are generated as time-correlated narrowband channels, $K = 0$ and $\sigma^2 = 1$. The resulting channel’s average delay spread and mean excess delay were 90.80 ns and 112.38 ns respectively.

In fig. 3, two graphics of the simulated wideband channel are depicted. Fig. 3.a) shows the average impulse response profile of the channel normalized with respect to the maximum received amplitude. In fig. 3.b), the channel’s impulse response profile is shown in the three-dimensional space, where the time variations of the rays’ amplitudes, which are typified by a Rayleigh pdf, can be noticed.

Since the distribution of rays over the excess delay axis doesn’t change, the impulse response profile is almost the same during the whole time observation interval; hence, this model can be employed for channels with low mobility scatterers.

The transfer function of the channel is shown in fig. 4 where its frequency selectivity is obvious. The channel’s time variability, as a result of employing our algorithm, can be observed in fig. 4. Given that the frequency correlation function of the channel depends on the delay spread, the channel model presented in this section provides users with the ability of simulating slow-variant channels with controlled time and frequency correlation properties.

V. TIME-VARYING WIDEBAND CHANNEL MODEL

Because of the variant nature of indoor channels, the propagation delay of rays tends to vary with time [10]. As a result of those variations an apparition and vanishing process of rays will be observed in the channel’s impulse response profile [11]. For the discrete channel model case, such process will produce changes in the way that rays are distributed over the excess delay axis so that the ray’s number will be changing in time.

With the aim of representing the process mentioned above, in [11] and [12], ray’s amplitudes (at a fixed delay) are supposed to vary as a half-period sine wave. In this section, we replace the half-period sine amplitudes by correlated Rayleigh amplitudes generated using the method of section III and taking advantage of some properties of equation (12) when $K = 0$ [6].

Two probability density functions model the ray’s apparition and vanishing process. One characterizes the temporal separation between the apparitions $t_a$ of ray $k$ and ray $k+1$ (14) and another one characterizes the lifespan of each ray $l_a$ (15).

$$f(t_a) = \frac{N_a}{\Delta l} \exp\left(-\frac{N_a \cdot t_a}{\Delta l}\right)$$  \hspace{1cm} (14)$$

$$f(l_a) = \frac{1}{\Delta l} \exp\left(-\frac{l_a}{\Delta l}\right)$$  \hspace{1cm} (15)$$

$N_a$ and $\Delta l$ are the average number of multipath components and the average lifespan time of each ray, respectively.
changing the values of the ray’s apparition and vanishing controlled by changing the value of the average delay spread of environments with high mobility scatterers. Rapidly in time; therefore, it can be employed in the simulation of selectivity of the channel are clearly depicted. As a result of the channel is shown, where the time variability and frequency separation can be observed. In fig. 6, the transfer function of the three-dimensional space where the apparition and vanishing process into the time-varying discrete time impulse response model developed in the previous section. For the simulation, $\Delta t = 0.95, N_{r} = 15$ and $\Delta t / N_{r} = 38$ (average temporal rays separation) [12]. The results are presented in figures 5 and 6 where the values of the average delay spread and average mean excess delay are 92.71 ns and 131.95 ns respectively.

In fig. 5, the channel’s impulse response profile is shown in the three-dimensional space where the apparition and vanishing process can be observed. In fig. 6, the transfer function of the channel is shown, where the time variability and frequency selectivity of the channel are clearly depicted. As a result of the apparition and vanishing process, this channel model varies rapidly in time; therefore, it can be employed in the simulation of environments with high mobility scatterers.

For this model, the channel’s frequency selectivity can be controlled by changing the value of the average delay spread while the channel’s time variability can be controlled by changing the values of the ray’s apparition and vanishing process parameters.

VI. CONCLUSIONS

In this paper, a straightforward method for generating correlated Rayleigh fading envelopes has been proposed. The recursive nature of the proposed algorithm reduces computational costs in comparison with algorithms based on the Cholesky decomposition of $N \times N$ correlation matrixes. In addition, the algorithm has been empirically evaluated under the phase/amplitude relation criterion, reaffirming the suitability of the proposed method.

Based on the algorithm proposed in section III, two time-variant wideband models for slow-fading indoor radio propagation channels have been developed in this paper. The model given in section IV can be employed in the simulation of channels with low-mobility scatterers whereas the model given in section V can be employed for simulating channels with high-mobility scatterers. For the last model, it is possible to represent different channel’s conditions by changing the parameters of the apparition and vanishing process.

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