Long-term Provisioning of Radio Resources Based on their Utilization in Dense OFDMA Networks

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***Abstract*—This paper presents long-term resource provisioning schemes for dense multi-cell OFDMA-based networks, where multiple cells with possibly overlapping coverage areas compete for the same set of resources. The optimization is done over the long term, being independent of the specific users connected to each cell but dynamic enough to follow significant variations of the traffic load. In this sense, we focus on the average resource utilization (RU) and propose schemes to minimize the maximum RU of all cells. Firstly, we consider orthogonal resource usage among cells. Optimal closed-form expressions for the long-term resource provisioning are derived for this case. Secondly, we assume that resources can be reused at non-overlapping cells. In this case, the resource provisioning is solved in two steps: *i*) the number of resources required per cell is obtained by discretizing the optimal solution of a convex problem, and then *ii*) the specific resources to be utilized by each cell are determined by using graph coloring. In contrast to previous works, graph coloring can be applied to get an implementable solution for any condition of the cell loads. Simulation results show a significant reduction of the maximum RU, which translates into an increase of the served traffic and a reduction of the packet delay, as compared to static resource provisioning schemes.**

***Index Terms*—multi-cell system, resource provisioning, long-term optimization, resource utilization, graph coloring.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) is the radio access technology employed in downlink transmission of 3GPP LTE and LTE-A systems [1], as well as in both uplink and downlink of IEEE 802.16m advanced WiMAX [2]. Furthermore, it is one of the major candidates for different use cases of future 5G systems [3]. In OFDMA-based networks, the intra-cell users are assumed to be orthogonal to each other and the primary source of interference is inter-cell interference (ICI). For that reason, due to the upcoming network densification and scarcity of spectrum, efficient resource provisioning schemes able to improve spatial reuse while avoiding dominant ICI in multi-cell scenarios are required.

Static frequency planning schemes are traditionally adopted in cellular systems. The simplest one is frequency reuse-n, where the total bandwidth is partitioned in n bands and different bands are assigned to neighbor cells to avoid dominant ICI. To further improve the spatial reuse while minimizing ICI impact, fractional frequency reuse (FFR) was proposed [4], where the total bandwidth is partitioned such that *i*) cell-edge users of neighbor cells are assigned to different frequencies and *ii*) frequencies assigned to cell-interior users can be reused. However, these schemes are static and independent of the traffic loads of the cells. So, static frequency planning schemes are not useful for dense and irregular deployments where the traffic load can vary drastically over space and time. In this regard, resource provisioning schemes capable of adapting to the varying average traffic load are needed, which, for simplicity of operation and implementability, should be performed in the *long-term* (i.e. should be independent of the specific users to be served at a concrete time instant).

Mathematically speaking, resource provisioning in multi-cell scenarios (also known as dynamic channel assignment problem [5]) is a combinatorial optimization problem that can be mapped into a *graph coloring* problem and is, therefore, NP-hard. In this line, graph-based approaches are proposed in [6] to allow dynamic FFR and distribute the resources in the long-term according to the per-cell traffic loads. Similarly, [7] exploits graph coloring to perform the resource provisioning in the short-term by adapting the resource allocation to the individual traffic loads of the concrete users being served. The drawback of all existing schemes based on graph coloring is that they employ a graph (which varies according to the per-cell traffic loads and the network deployment) that might not be colorable with the number of colors (radio resources) available. This means that proper coloring[1] may not be possible or, equivalently, that an implementable solution satisfying the coloring constraints may not be found.

In this paper, we present a novel approach for long-term resource provisioning based on optimizing the *resource utilization* (RU), which measures the occupancy of a cell and is given by the ratio between the amount of data traffic and the effective capacity per cell. We formulate the resource provisioning strategy as a convex problem so as to minimize the maximum RU. When orthogonal resource usage among cells is considered, closed-form expressions are derived. On the other hand, when reuse of resources among non-overlapping cells is allowed, we propose a scheme that works as follows: first, the problem is formulated according to the interference graph (where we impose that neighbor cells must use different resources to avoid strong ICI); then, the optimal distribution per cell is obtained by solving the problem; finally, such repartition is mapped into real resources by using graph coloring over a new extended graph. Differently from previous works ([6][7]), we first solve an optimization problem which ensures that proper coloring over the new extended graph will be possible.

[1] A proper coloring of a graph is an assignment of colors to the vertices of the graph such that no adjacent vertices have the same color.

II. System Model

Consider a downlink cellular network composed of K randomly and densely distributed cells ($k = 1, \dots, K$). Due to the random and dense geographical distribution, the coverage area of some cells may overlap, i.e. some users might be in a location covered by multiple cells. So, if these cells use the same frequency-time resource, interference will be generated.

Assume OFDMA, as in LTE-A downlink [1]. There are N effective frequency sub-channels, also called resource blocks (RBs) in LTE-A, available in the system each with the same bandwidth. No power control in downlink is assumed such that the transmit power per resource (or RB) is fixed or, equivalently, a spectral power mask is adopted.

Our objective is to provide a method to decide the resources assigned to each cell in the *long term*[2], i.e.:

- it should not be adapted to a concrete set of users, as this would require to change the amount of resources whenever a user appears o disappears in the cell, but
- it should be dynamic enough to adapt to significant changes in the average traffic load.

In addition, the proposed method must seek to: *i*) use the spectrum resources efficiently by avoiding over-provisioning of resources, *ii*) avoid situations of high resource occupancy that will lead to high packet delays, and *iii*) provide quality-of-service (QoS) to the users of the different cells. A suitable measure that captures these requirements is the RU.

The strategy for long-term provisioning proposed in this paper is performed at a central controller that controls the K cells and disposes of long-term information and network topology information. As long-term information, a single parameter per cell is required. As network topology information we refer to knowledge of the interference graph [6].

The *interference graph* ($\mathcal{G}_I$) is constructed as follows:

- every cell in the network defines a vertex and
- any two cells (represented through a vertex) in an interference situation are connected with an edge.

Interference situations appear whenever the transmission of one cell could potentially interfere the transmission of another cell if the same resource is used. For example, in one-tier networks, an interference situation could be established when two cells were closer than a threshold distance.

III. Resource Utilization

The RU is a measure widely used in 3GPP evaluations to report the percentage of resources employed by a cell [1]. It is given by the ratio between the total number of resources used over the total number of resources available for data traffic.

The RU of the k-th cell (ρ_k) can be estimated as the average traffic load of the cell (in bits/s) over the amount of traffic that the cell can serve (i.e. the effective capacity, in bits/s). It is also referred to as the normalized load of a cell, see [8]:

$$\rho_k = \frac{\lambda_k \mathbb{E}\{L_k\}}{x_k C_k} = \frac{\alpha_k}{x_k}, \tag{1}$$

where $\alpha_k = \lambda_k \mathbb{E}\{L_k\}/C_k$, λ_k is the mean packet arrival rate at the k-th cell (in packets/s), $\mathbb{E}\{L_k\}$ denotes the mean packet length (in bits/packet), x_k refers to the number of resources assigned to the k-th cell, and C_k is the average spectral efficiency of the k-th cell (in bits/s/resource).

It can be observed in (1) that increasing the amount of resources at the k-th cell (x_k) leads to a low ρ_k and hence an inefficient usage of resources (as they could be used for other purposes). On the contrary, reducing the amount of resources (x_k) leads to a high ρ_k, which increases the packet delay and reduces the QoS of the users in the k-th cell.

Note that, although the RU is a measure bounded between 0 and 1, the normalized load of the cell ρ_k in (1) could be larger than 1 by definition if the traffic load is very high. However, systems should be properly designed so as not to "blow up" the queues, i.e. the resource provisioning strategy should assure that $\rho_k \leq 1$. Otherwise, if $\rho_k > 1$, the system would be unstable, inducing large queue sizes, losses of packets, and unacceptable packet delays. From now on we assume that the system will operate under this condition ($\rho_k \leq 1$), in which the normalized load ρ_k is equivalent to the RU of the cell.

Parameters $\alpha_k, \forall k$, in (1) correspond to the long-term information required at the central controller to perform the optimization. α_k can be estimated based on the average traffic load ($\lambda_k \mathbb{E}\{L_k\}$) and the average spectral efficiency (C_k) of the k-th cell. The later, C_k, can be estimated based on the statistics of the previously served transmission rates. There are two main approaches in the literature to estimate C_k, which depend on the scheduling strategy that is adopted. In case a round-robin scheduler is used, C_k is given by:

$$C_k = \frac{1}{M} \sum_{i=1}^{M} R_{i,k}, \tag{2}$$

where $R_{i,k}$ is the instantaneous transmission rate given to user i in cell k when such user is being served (in bits/s/resource) and M denotes the number of users among which the statistics are computed. Alternatively, if the scheduler is such that gives the same rate to all users, C_k can be estimated as [9]:

$$C_k = M \left(\sum_{i=1}^{M} \frac{1}{R_{i,k}} \right)^{-1}. \tag{3}$$

If packet arrival instants are modeled as a Poisson process, for $\rho_k \leq 1$, the average number of bits in the queue of the k-th cell (W_k) is related to ρ_k in (1) as follows [8]:

$$W_k = \frac{\rho_k}{1-\rho_k} \frac{\mathbb{E}\{L_k^2\}}{2\mathbb{E}\{L_k\}}. \tag{4}$$

Clearly, longer queue sizes imply higher packet delays. Therefore, in a single-cell system minimizing the RU is equivalent to minimize the average number of bits in the system (or to minimize the average packet delay), see (4).

[2] Long-term resource provisioning involves provisioning of frequency resources for several consecutive transmission time intervals (TTIs). Rather than dynamically changing the resource provisioning at each TTI, long-term resource provisioning is preferable for simplicity of operation/implementation.

In a multi-cell scenario, where multiple cells compete for the same set of resources, an efficient resource provisioning is such that the resources are distributed among all the cells in a balanced way trying to avoid very different occupancies for different cells. In this sense, a suitable optimization criterion is the minimization of the maximum RU (max RU) among the cells, such that resources are fairly distributed and more resources are given to those cells with larger traffic loads and/or those cells experiencing greater delays (see (1)).

Finally, recall that the resource provisioning in multi-cell environments is a combinatorial optimization problem that involves high complexity [5]. For that reason, we focus on solving a relaxed version of the optimization problem with continuous variables (corresponding to the distribution of resources, $\{x_k\}$) and then the obtained result is discretized.

IV. Orthogonal Resource Provisioning

In this section, we derive a long-term resource provisioning scheme when orthogonal resource usage among cells is assumed (i.e. the spectrum is split into disjoint sets and each one is assigned to one cell).

A. Problem formulation and resolution

The minimization of the max RU ($\rho_k = \frac{\alpha_k}{x_k}$) subject to the constraint that the sum of all resources must be lower or equal to the total number of resources is formulated as:

$$(\mathrm{P}_0): \underset{\{x_k\}}{\text{minimize}} \; \max_k \left(\frac{\alpha_k}{x_k}\right) \quad \text{subject to} \sum_{k=1}^{K} x_k \leq N, \tag{5}$$

where N is the total number of resources. Problem (P_0) in (5) is convex w.r.t. $\{x_k\}$. In the following we derive the optimal solution in closed-form.

Problem (P_0) in (5) may be equivalently written as:

$$(\mathrm{P}_1): \underset{\{x_k\},t}{\text{minimize}} \; t \quad \text{subject to} \begin{cases} \frac{\alpha_k}{x_k} \leq t \quad \forall k \\ \sum_{k=1}^{K} x_k \leq N, \end{cases} \tag{6}$$

which is jointly convex w.r.t. $\{x_k\}$ and t. The optimal solution to (P_1) in (6) is such that all constraints are satisfied with equality [10]. This means that all RU are equal: $\frac{\alpha_k}{x_k} = \rho$. Therefore, the optimal solution for $\{x_k\}$ in (P_1) (and, consequently, in (P_0)) has the following structure:

$$x_k^\star = \frac{\alpha_k}{\rho}. \tag{7}$$

By including (7) into the last constraint of (P_1) in (6) we have: $\sum_{k=1}^{K} \frac{\alpha_k}{\rho} = N$, from which ρ can be isolated. Then, the optimal solution for the resource provisioning under orthogonal resource usage and minimum max RU is:

$$x_k^\star = \frac{\alpha_k}{\sum_{l=1}^{K} \alpha_l} N. \tag{8}$$

The optimal solution in (8) gives more resources to those cells experiencing a higher ratio among the average traffic load and the average spectral efficiency, i.e. higher α_k.

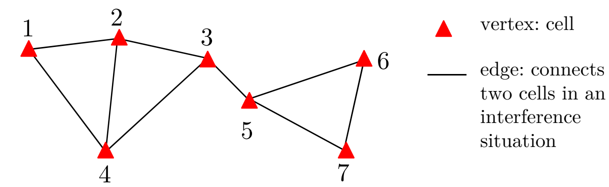

Fig. 1: Interference graph for a network of K=7 cells.

B. Mapping the solution into real resources

The optimal resource provisioning has been found as a continuous distribution of the total available spectrum. However, practical systems dispose of an integer number of resources to be allocated. So we should convert the optimal distribution into a discrete number of resources per-cell satisfying the constraint in (5): $\sum_{k=1}^{K} x_k \leq N$, and then map it into real resources. The round down (i.e. take the floor) would always satisfy the constraint. But any other rounding that satisfies the constraint is a valid one. Afterwards, mapping the number of resources per-cell into real resources is straightforward.

V. Graph-Based Resource Provisioning

In this section, we derive a long-term resource provisioning scheme when reuse of resources among cells that have non-overlapping coverage areas is permitted. Similar criterion as in Section IV is adopted, but now we will impose orthogonality only among the sets of resources assigned to cells in an interference situation. To do so, we use information given by the *interference graph* $\mathcal{G}_I$ (see Section II) to formulate the problem and then exploit graph coloring to solve it.

A. Problem formulation and resolution

Based on the interference graph $\mathcal{G}_I$, we impose that neighbor cells (those connected through an edge in the interference graph) cannot reuse resources. This is done through the inclusion of proper constraints into the optimization problem. We define an orthogonality constraint for each maximal clique[3] of $\mathcal{G}_I$, where each constraint includes the resources of all the vertices that conform each maximal clique.

As an example, Fig. 1 shows the interference graph in a network composed of 7 cells. In this case, the orthogonality constraints derived from the interference graph in Fig. 1, which will be represented by $\mathcal{C}$ in what follows, are:

$$\mathcal{C} = \left\{ \begin{array}{ll} x_1 + x_2 + x_4 \leq N, & x_2 + x_3 + x_4 \leq N, \\ x_3 + x_5 \leq N, & x_5 + x_6 + x_7 \leq N \end{array} \right\}. \tag{9}$$

This set of orthogonality constraints defines a necessary condition for an implementable solution to exist (i.e. any valid assignation of resources that can be satisfied without conflicts in $\mathcal{G}_I$ will satisfy the constraint set). Moreover, as we will see in subsection V-B, said constraint set defines a necessary and sufficient condition for an implementable solution to exist and to be found if $\mathcal{G}_I$ has certain properties. Otherwise, $\mathcal{G}_I$ can be

[3] A maximal clique of a graph $\mathcal{G}$ is a complete subgraph of $\mathcal{G}$ that cannot be extended by including one more adjacent vertex, meaning it is not a subset of a larger complete subgraph of $\mathcal{G}$. Algorithms to find all the maximal cliques of a graph are available in the literature (e.g. in [11]).

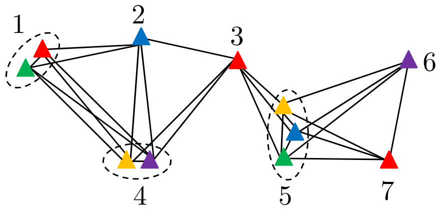

Fig. 2: Extended graph of the interference graph in Fig. 1. K=7 cells, N=5 resources, $x_1^\star = 2$, $x_2^\star = 1$, $x_3^\star = 1$, $x_4^\star = 2$, $x_5^\star = 3$, $x_6^\star = 1$, $x_7^\star = 1$.

slightly modified by adding some edges to meet the properties and hence ensuring the sufficient condition.

According to the constraint set, we define the optimization problem by following the minimization of the maximum RU ($\rho_k = \frac{\alpha_k}{x_k}$) subject to the constraints extracted from $\mathcal{G}_I$:

$$(\mathrm{P}_2): \ \underset{\{x_k\}}{\text{minimize}} \ \max_k \left(\frac{\alpha_k}{x_k} \right) \quad \text{subject to} \ \ \mathcal{C}. \tag{10}$$

The objective function of (P_2) in (10) is convex w.r.t. $\{x_k\}$. Then, as the constraint set $\mathcal{C}$ is linear (and hence convex) on the optimization variables, problem (P_2) in (10) is convex w.r.t. $\{x_k\}$. Although a closed-form solution for resource provisioning cannot be obtained, problem (P_2) in (10) can be solved in polynomial time using convex optimization tools as, for instance, interior point methods [10]. In the following, let us denote by $\{x_k^\star\}$ the optimal solution to (P_2) in (10).

B. Mapping the solution into real resources

Once the optimal continuous distribution of spectrum to be assigned to each cell is obtained, $\{x_k^\star\}$, we should map it into real resources. First, rounding the optimal distribution into an integer number of resources per cell is required. Any rounding within the constraint set $\mathcal{C}$ in (10) is a valid one. After the rounding, we should determine which are the specific resources to be used by each cell, i.e. we should map the number of resources per cell into real resources. Multiple mappings may exist, but the key point is to guarantee that there exists at least one implementable mapping. To obtain the mapping, we define an extended graph and then use graph coloring to color it (where each color represents a resource).

The *extended graph* ($\mathcal{G}_E$) is built as follows:

- for each k-th cell, as many vertices as the number of resources obtained from discretizing the optimal solution to problem (P_2) in (10) (i.e. $x_k^\star$) are included, and
- all the vertices corresponding to the k-th cell have edges to all vertices corresponding to all other cells that were connected in the interference graph $\mathcal{G}_I$ to the k-th cell.

Note that $\mathcal{G}_E$ is formed through the replication of vertices in $\mathcal{G}_I$ according to the discretized solution of problem (P_2) in (10). An example of the extended graph corresponding to the interference graph in Fig. 1 is shown in Fig. 2 assuming that $N = 5$ resources were available for optimization of problem (P_2) in (10) and that the obtained discretized solution was: $x_1^\star = 2$, $x_2^\star = 1$, $x_3^\star = 1$, $x_4^\star = 2$, $x_5^\star = 3$, $x_6^\star = 1$, $x_7^\star = 1$. Then, we apply graph coloring over the extended graph.

Graph coloring is an NP-hard problem for arbitrary graphs. In addition, most of the coloring algorithms were proposed to color a graph under the assumption that there are enough colors to color the graph. When the number of colors is limited to N, *proper N-coloring*[4] may not be possible. This is the reason why proper N-coloring cannot be ensured in recent resource provisioning schemes provided in the literature (either in the long-term [6] or in the short-term [7]): they use a graph that may not have a proper N-coloring due to the randomness in the traffic loads and in the network deployment, see [6][7].

However, a well-known fact about graph coloring is that the chromatic number[5] of a graph $\mathcal{X}(\mathcal{G})$ is lower bounded by the clique number[6] of the graph $\omega(\mathcal{G})$: i.e. $\mathcal{X}(\mathcal{G}) \geq \omega(\mathcal{G})$, being the bound tight for perfect graphs[7] [12][13]. In our case, the clique number of the extended graph is exactly N: $\omega(\mathcal{G}_E)$=N, as it was imposed in the constraint set (see (9)). Therefore, as perfection is preserved through replication of vertices [13], if $\mathcal{G}_I$ is a perfect graph then $\mathcal{G}_E$ will be a perfect graph with $\mathcal{X}(\mathcal{G}_E) = \omega(\mathcal{G}_E) = N$ such that $\mathcal{G}_E$ will have a proper N-coloring. So, for perfect graphs, the proposed constraint set is a necessary and sufficient condition for an implementable solution to exist. Note that the interference graph in Fig. 1 and its extended graph in Fig. 2, as well as the interference graph used for simulations in Fig. 4, are perfect graphs.

***Lemma** 1:* After determining the number of resources required per cell by discretizing the optimal solution to problem (P_2) in (10) with N available resources, proper N-coloring of $\mathcal{G}_E$ is possible (i.e. an implementable mapping into real resources exists) if $\mathcal{G}_I$ is a perfect graph.

Proof: The clique number of $\mathcal{G}_E$ is $\omega(\mathcal{G}_E) = N$, as it is imposed by the constraints to problem (P_2) in (10) where N resources are available (see (9)). Under the assumption that $\mathcal{G}_I$ is a perfect graph (and, by replication, also $\mathcal{G}_E$ is), then: $\mathcal{X}(\mathcal{G}_E) = N$. So proper N-coloring of $\mathcal{G}_E$ is possible. ■

Perfect graphs can be recognized in polynomial time [14]. There are many classes of perfect graphs, see [15] where up to 120 classes are described. Moreover, any graph can be transformed into a chordal graph (a class of perfect graphs) by adding some edges such that the graph possesses no cycles of length ≥ 4 [16]. Therefore, in case $\mathcal{G}_I$ is not a perfect graph, we can transform it into a perfect graph, define the constraint set accordingly, and hence ensure the sufficient condition.

Finally, optimal coloring of perfect graphs can be obtained with Greedy algorithms in polynomial time [17, Sect. 9]. Therefore, an implementable solution does not only exist but can also be found for any conditions of the cell loads. For example, we can color $\mathcal{G}_E$ with $\mathcal{X}(\mathcal{G}_E) = N$ colors by applying well-known low-complexity algorithms (e.g. [18]).

[4] A proper N-coloring of a graph is an assignment of colors to the vertices of the graph such that no adjacent vertices have the same color when N colors are available.

[5] The chromatic number of a graph $\mathcal{G}$, denoted by $\mathcal{X}(\mathcal{G})$, is the minimum number of different colors required for a proper coloring of the graph.

[6] The clique number of a graph $\mathcal{G}$, denoted by $\omega(\mathcal{G})$, is the cardinality of the largest maximal clique of the graph.

[7] A graph $\mathcal{G}$ is a perfect graph if and only if neither $\mathcal{G}$ nor its complement have an odd-length induced cycle of length 5 or more [12].

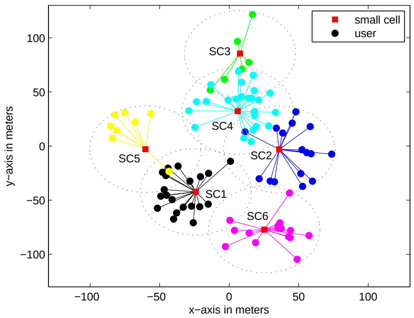

Fig. 3: Example of network deployment used for simulations with $K = 6$ SCs. The number of users at each SC is: $20, 16, 6, 24, 8, 15$, respectively.

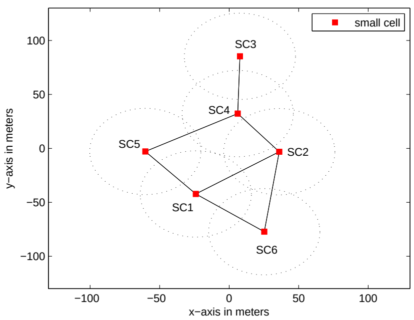

Fig. 4: Interference graph associated to the network deployment in Fig. 3. The threshold distance is 80 m.

VI. Simulation Results

The scenario consists of a deployment of $K = 6$ small cells (SCs), which are randomly placed within a circular area of 100 m radius with a minimum distance of 40 m among them. I_k users are randomly placed around each k-th SC in a concentric 40 m radius circle with a minimum distance between users and SC of 10 m. All SCs operate on the same carrier frequency at 2 GHz with 10 MHz bandwidth (BW), where $N = 50$ RBs are available. Path loss and shadowing models follow specifications in [19] for multi-cell pico scenario (see Table I). Downlink transmission is evaluated. The antenna pattern is omnidirectional and the transmit power is 24 dBm at SC. Noise spectral density is -174 dBm/Hz.

As the objective of the following simulations is to show the benefits of the proposed long-term resource provisioning schemes when the load varies per SC, the number of users associated to each SC (I_k) is set to $20, 16, 6, 24, 8, 15$ users, respectively. A deployment example is shown in Fig. 3.

The non-full buffer FTP3 traffic model [20] is used for traffic generation, where packets for the same user arrive according to a Poisson process with arrival rate λ (in packets/s) and the packet length is fixed and equal to $L = 0.5$ Mbits (i.e. $\mathbb{E}\{L_k\} = L$, $\forall k$). Therefore, the mean packet arrival rate for the k-th SC is proportional to the number of users, i.e. $\lambda_k = \lambda I_k$ in (1), and differs among SCs.

Results are averaged over 1000 random deployments of the users but, for simplicity, the SCs' positions are kept fixed. We assume a threshold distance equal to 80 m. Hence, the interference graph associated to the deployment in Fig. 3 is the one depicted in Fig. 4, from which the set of orthogonality constraints for the graph-based reuse scheme are:

$$\mathcal{C}=\left\{ \begin{array}{l} x_1+x_2+x_6 \leq 50, \quad x_2+x_4 \leq 50, \\ x_1+x_5 \leq 50, \;\; x_4+x_5 \leq 50, \;\; x_3+x_4 \leq 50 \end{array} \right\} \quad (11)$$

1000 frames of 10 ms are simulated, being the simulation time equal to 10 s. On each frame, the active users per SC (i.e. users with packets in the queues) are uniformly distributed among the available RBs. The number of available RBs per SC is determined in the long-term based on the different schemes.

The following schemes are evaluated:

- **orthogonal uniform**: the 50 RBs are orthogonally distributed among the 6 SCs in an almost uniform manner.
- **orthogonal maxRU**: the 50 RBs are orthogonally distributed among the 6 SCs according to the proposed resource provisioning in Section IV.A (see (P_0) in (5)). The distribution follows the closed-form solution in (8).
- **full reuse**: the 50 RBs can be employed by all the 6 SCs whenever they have packets to be transmitted.
- **frequency reuse-3**: frequency planning scheme with reuse factor equal to 3, i.e. the 50 RBs are split into 3 disjoint sets and neighboring SCs use different sets.
- **graph-based reuse maxRU**: the 50 RBs are distributed among the 6 SCs according to the proposed resource provisioning in Section V with the constraints imposed by the interference graph in (11) (see (P_2) in (10)).

In the proposed schemes based on RU optimization, i.e. 'orthogonal maxRU' and 'graph-based reuse maxRU', the average spectral efficiency is assumed to be equal for all SCs ($C_k = C, \forall k$) as all SCs dispose of the same power, users are uniformly distributed in space, and a round-robin scheduler is adopted. Therefore, parameter α_k in (1) is given by:

$$\alpha_k = \frac{\lambda I_k L}{C}. \quad (12)$$

Fig. 5 shows the maximum RU versus λ (in packets/s). The maximum RU corresponds to the average over deployments of the maximum RU among the 6 SCs. It is important to recall that the RU is computed over the available resources, which is different on each scheme. It can be observed that 'graph-based reuse maxRU' reduces significantly the max RU as compared to all the other schemes, except for very low traffic loads ($\lambda = 0.5$) where 'full reuse' is better. 'orthogonal maxRU' reduces the max RU as compared to 'orthogonal

TABLE I: Simulation Parameters.

Parameter	Value
SCs deployment	$K = 6$ SCs, circular area 100 m radius
Users deployment	$I_k = [20, 16, 6, 24, 8, 15]$ users per SC
Carrier frequency	2 GHz
BW, Number of RBs	10 MHz, $N = 50$ RBs
SC transmit power	24 dBm
Pathloss	LOS: 103.8+20.9$\log_{10}(d)$, d: distance in Km, NLOS: 145.4+37.5$\log_{10}(d)$
Traffic model	FTP3. Poisson process with arrival rate per-user λ. Packet length $L = 0.5$ Mbits
Scheduler	round-robin
Link adaptation	ideal

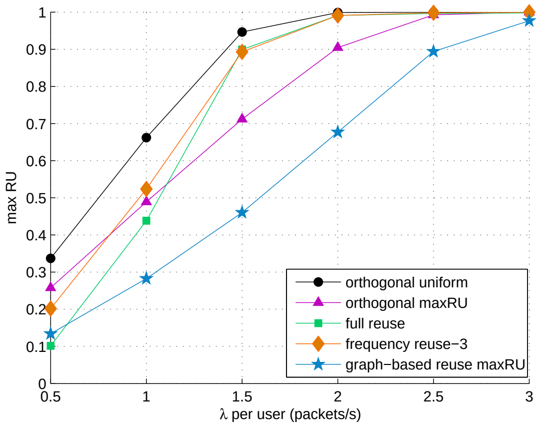

Fig. 5: Maximum RU versus packet arrival rate per user (λ, in packets/s).

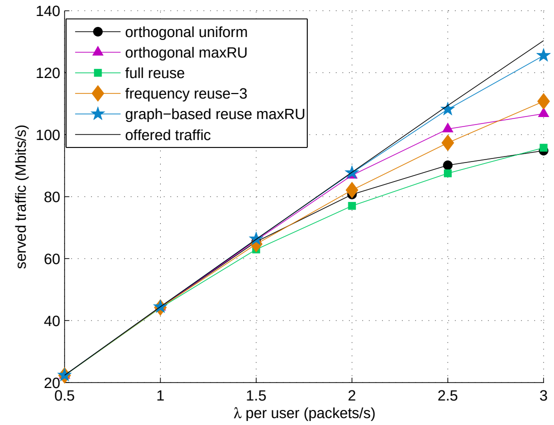

Fig. 6: Served traffic (in Mbits/s) versus packet arrival rate per user (λ, in packets/s).

uniform' and, as the traffic load increases (i.e. as λ increases), it also improves 'full reuse' and 'frequency reuse-3'. This is because *i*) in 'full reuse' there is an increased level of interference that cannot be controlled and *ii*) in 'frequency reuse-3' the resources are equally distributed, which make these schemes being saturated (in terms of occupancy) earlier than 'orthogonal maxRU' as λ increases. Therefore, resources are fairly allocated with the proposed schemes based on RU optimization because the distribution takes into account the per-cell traffic loads and spectral efficiencies such that all SCs have a similar RU. To conclude, the scheme that provides the lowest max RU and can work in a large range of traffic loads without being saturated is the proposed 'graph-based reuse maxRU' due to the efficient reuse and distribution of resources where strong interference conditions are avoided.

Fig. 6 displays the total traffic served by each scheme (in Mbits/s) versus λ (in packets/s). The total offered traffic (in Mbits/s) is also shown with a black line. All orthogonal schemes ('orthogonal uniform' and 'orthogonal maxRU') get saturated in terms of served traffic as λ increases because the number of resources per SC is low and all SCs are highly occupied for high values of λ. The remaining schemes ('full reuse', 'frequency reuse-3', and 'graph-based reuse maxRU') have already not reached the maximum occupancy at all the SCs (although maybe in some SCs) with $\lambda = 3$ packets/s. However, the proposed 'graph-based reuse maxRU' scheme allows serving the maximum quantity of traffic as λ increases. Note that 'full reuse' and 'orthogonal uniform' provide the lowest performance in terms of served traffic at high traffic loads, because: *i*) with 'full reuse', high interference is present in the dense scenario under consideration (see Fig. 3) if no interference coordination is performed (thus low achievable rates are obtained), and *ii*) with 'orthogonal uniform', interference is avoided but there are not enough resources per SC to allocate the amount of offered traffic.

Fig. 7 depicts the mean packet delay (in s) versus λ (in packets/s). The packet delay is computed from the moment a packet arrives at the queue to the moment in which the transmission of the whole packet is completed. To compute

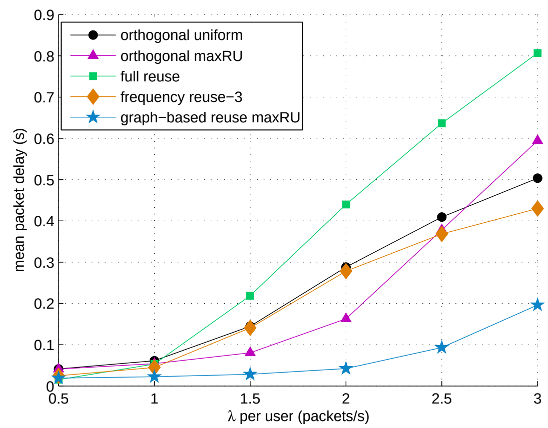

Fig. 7: Mean packet delay (in s) versus packet arrival rate per user (λ, in packets/s).

the statistics, only packets whose transmission is completed during the simulation time (i.e. 10 s) are considered. It can be observed that mean packet delays are reduced with the proposed RU-based schemes. In particular, the 'graph-based reuse maxRU' scheme is the one providing a significantly lower value of the mean packet delay for all the simulated packet arrival rates (λ).

Therefore, although we have focused on minimizing the RU of all cells, it can be concluded from Fig. 6 and Fig. 7 that said optimization is effectively translated into an increase of the served traffic and a reduction of the mean packet delay in dense multi-cell scenarios.

Finally, let us show the dispersion of the RU and the packet delay for a fixed value of λ where the system is not saturated. We chose $\lambda = 1.5$ packets/s according to results in Fig. 5.

Fig. 8 displays the cumulative density function (CDF) of the RU of all SCs in the network for $\lambda = 1.5$ packets/s. It can be observed that the dispersion of RU values with 'orthogonal uniform', 'full reuse', and 'frequency reuse-3', is very large. On the contrary, the proposed 'orthogonal maxRU' and 'graph-based reuse maxRU' allow having the RU of all SCs in a reduced range of values, i.e. the variability in the occupancy of the cells is much low, thanks to the efficient distribution of

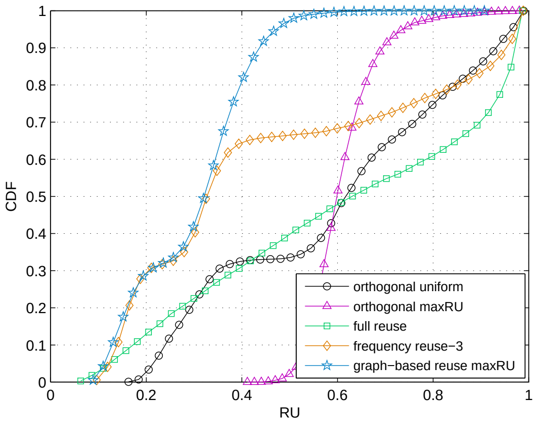

Fig. 8: CDF of RU of all SCs for $\lambda = 1.5$ packets/s.

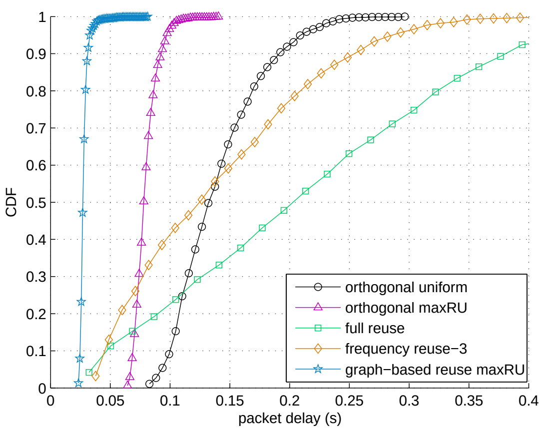

Fig. 9: CDF of the packet delay (in s) for $\lambda = 1.5$ packets/s.

frequency resources. Note also that 'orthogonal maxRU' has a lower dispersion in terms of RU than 'graph-based reuse maxRU'. This is because the optimal solution to (P_0) in (5) corresponds to equal RU for all cells (see (7)), however, the optimal solution to (P_2) in (10) might not lead to equal RU for all cells due to the constraint set (i.e. multiple optimal solutions leading to the same objective function and satisfying the constraint set could be obtained in general). Even though, the maximum RU value is effectively reduced with 'graph-based reuse maxRU'.

Fig. 9 depicts the CDF of the packet delay (in s) for $\lambda = 1.5$ packets/s. The dispersion of the packet delays is much lower in the proposed RU-based schemes than in static resource provisioning approaches since we aim at distributing the resources to get similar RU among cells and, as consequence, similar packet delays are obtained (see formula for average number of bits in the queue in (4), which is related to the packet delay). Differently from the RU results (see Fig. 8), the dispersion in terms of packet delay is similar with 'orthogonal maxRU' and 'graph-based reuse maxRU' because in 'graph-based reuse maxRU' those cells increasing the RU dispersion as compared to 'orthogonal maxRU' (i.e. those cells having a low RU) can not involve a reduction of the packet delay lower than the minimum required for a complete packet transmission.

VII. Conclusions

This paper proposes long-term resource provisioning schemes based on optimization of the RU factors, or occupancies, of the cells for dense multi-cell OFDMA-based networks. When orthogonal resource usage among cells is assumed, the optimal distribution of resources per cell is obtained in closed-form. In case of allowing to reuse resources among cells, we present a graph-based reuse scheme whereby the optimization problem is set according to the interference graph, then it is optimally solved, and finally graph coloring is applied to map the obtained solution into real resources. Simulation results show that the maximum RU, the total amount of served traffic, and the mean packet delay are significantly improved with the proposed graph-based reuse scheme at all the simulated traffic load conditions, as it allows an efficient reuse and distribution of resources that avoids strong interference conditions.

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