Subset relay selection in wireless cooperative networks using sparsity-inducing norms

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***Abstract*—This paper addresses the problem of multiple relay selection in a two-hop wireless cooperative network. In particular, the proposed technique selects the best subset of relays, in a distributed beamforming scheme, which maximizes the signal-to-noise ratio at the destination node subject to individual power constraints at the relays. The selection of the best subset of K relays out of a set of N potential relay nodes, under individual power constraints, is a hard combinatorial problem with a high computational burden. The approach considered herein consists in relaxing this problem into a convex one by considering a sparsity-inducing norm. The method exposed in this paper is based on the knowledge of the second-order statistics of the channels and achieves a near-optimal performance with a computational burden which is far less than the one needed in the combinatorial search. Furthermore, in the proposed technique, contrary to other approaches in the literature, the relays are not limited to cooperate with full power.**

***Index Terms*—Relay selection, distributed beamforming, semidefinite relaxation, sparsity-inducing norms.**

I. Introduction

The use of relays has been widely analyzed in the signal processing literature as a mechanism to increase the reliability and the spatial diversity of wireless communication systems, resulting in wider coverage, higher rates and lower transmit power compared to point-to-point links. The simplest wireless relay network consists of a single source-destination pair and N relay nodes which cooperate with the source to send the message to the destination. In this context, distributed relay beamforming [1], [2], a.k.a. network beamforming, is a powerful tool which provides directional gain and power efficiency. In this technique the relays cooperate adjusting their transmission weights in order to form a virtual beam to the destination, mitigating the effect of fading channels and unwanted interferences. Since relay nodes have particular constraints on the power of their batteries, the maximization of the received signal-to-noise ratio (SNR) at the destination, subject to per-relay power constraints, has deserved special attention in the literature [2]. Interestingly enough, to achieve the maximum SNR at the destination, each relay may not transmit at the maximum allowable power.

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In a wireless relay networks power optimization is a major issue and relay selection is of particular interest to save power and to increase the lifetime of the batteries. The goal of relay selection is to choose one or more nodes for the cooperation with the source. Most of the approaches in the literature are focused on the selection of the best relay for the cooperation, e.g. [3]. Nonetheless, Single Relay Selection (SRS) do not fully exploit the available degrees of spatial diversity and in adverse communications environments, transmitting over a single relay may not be sufficient to achieve a desired error performance. On the contrary, a high number of cooperating nodes could cause a negative influence over the bandwidth efficiency. This has motivated the generalization of the SRS idea, allowing more than one relay to cooperate in the retransmission. A good trade-off between the band efficiency and the error performance can be achieved if multiple relays are selected for the cooperation instead of only one node or all the potential relays [4]. This idea is the so-called Multiple Relay Selection (MRS) and has attracted interest in several references, e.g. [4], [5], [6]. In all these techniques, the relays are not allowed to adjust their power arbitrarily, i.e. each relay has only two choices to cooperate with full power or not cooperate at all. Furthermore, these approaches are based on the knowledge of the instantaneous Channel State Information (CSI) of the channels of the wireless relay network and, unfortunately, this assumption is often violated in practical scenarios.

This paper proposes a new technique which deals with the problem of multiple relay selection in a two-hop wireless cooperative network under individual power constraints in the relay nodes. This method selects the best subset of relays for retransmission and the corresponding weights which maximize the SNR at the destination node. The proposed technique, in contrast to other approaches in the literature, is based on knowledge of the second-order statistics of CSI and the relays are not limited to cooperate with full power. Actually, this optimization leads to solutions in which the selected subset of relays may not use their maximum allowable power. Whereas the problem of selecting the best subset of K out of N potential relays and their corresponding weights is a hard combinatorial problem, it can be relaxed into a convex problem with an affordable computational complexity. This is done by means of a sparsity-inducing norm, the l_1-norm squared [7], a surrogate of the cardinality operator which promotes zeros in the final solution vector. The proposed technique achieves a near-optimal performance with a computational burden which is significantly lower than the one of the exhaustive search.

The paper is structured as follows. Section II presents the relay network model and introduces the multiple relay selection problem. The proposed technique for the selection of the subset of cooperating nodes is presented in Section III. Numerical results are shown in Section IV. Finally, Section V provides some concluding remarks.

II. System model and problem formulation

Consider a two-hop wireless cooperative network which consists of a single source-destination pair and N potential relays which cooperate with the source to send the message to the destination node as shown in Fig. 1. Each of the nodes in this scheme is equipped with a single antenna and a perfect synchronization between all the terminals is assumed. Furthermore, the direct link between the source and the destination is ignored due to the poor quality of the channel between both terminals. The channel gains between the source and the ith relay and the between the ith relay and destination, denoted by h_i and g_i respectively, are considered as random values. Similar to [2] and [8], the joint second-order statistics of these channels is assumed to be known at a central node, for instance, the destination, which estimates the optimal weights based on past measurements and then delivers them to the relay nodes via a dedicated channel.

In the scheme considered in this paper the communication takes place in two slots. During the first slot the source transmits the signal $\sqrt{P_s}s$ to the relays, being s the information symbols and P_s the source transmit power. For the sake of simplicity, it is assumed that $E\{|s|^2\} = 1$. The received signal at the ith relay is

$$x_i = \sqrt{P_s} h_i s + \eta_i \tag{1}$$

being η_i the additive noise at the ith relay which has a known variance σ_r^2. In the second slot each relay multiply the received signal by a complex weight w_i and transmits $z_i = w_i x_i$. Thus, the received signal at the destination can be expressed as

$$z = \underbrace{\sqrt{P_s}\sum_{i=1}^{N} w_i h_i g_i s}_{\text{signal of interest}} + \underbrace{\sum_{i=1}^{N} w_i g_i \eta_i + n_d}_{\text{Total noise}} \tag{2}$$

where n_d is the noise at the destination which has a zero mean and known variance σ_d^2.

A. Multiple relay selection and SNR maximization under per-relay power constraints

Consider the joint problem of selecting the best subset of K relay nodes out of a set of N potential relays and the estimation of the weights which maximize the SNR at the destination under individual constraints on the power of the relays. As it will presented later, the reduction of the number of cooperating relays implies a reduction in the total power transmitted by the relays and this has a result an increase on

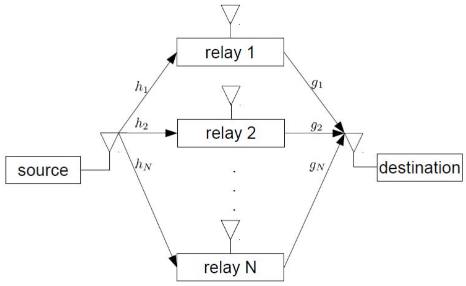

Fig. 1. Wireless relay network

the lifetime of the batteries of the cooperating nodes. This problem is given by

$$\begin{aligned} \max_{\mathbf{w}} \quad & \text{SNR} \\ \text{s.t.} \quad & p_i \le P_i \qquad \forall i = 1, ..., N \\ & \text{card}(\mathbf{w}) = K \end{aligned} \tag{3}$$

where $\mathbf{w} = [w_1 \dots w_N]^T$is the network beamforming vector and card$(\cdot)$ denotes the cardinality operator which returns the number of non-zero coefficients of its argument. P_i and p_i are maximum allowable transmit power and the actual transmit power of the ith relay, respectively.

Let us derive the mathematical expressions for the SNR and p_i. The power of the signal component is given by

$$P_d = E\left\{\left|\sqrt{P_s}\sum_{i=1}^{N} w_i h_i g_i s\right|^2\right\} = \mathbf{w}^H \mathbf{R} \mathbf{w} \tag{4}$$

being $\mathbf{R} = P_s E\left\{(\mathbf{h} \odot \mathbf{g})(\mathbf{h} \odot \mathbf{g})^H\right\}$. In this expression, the operator $\odot$ represents the Schur-Hadamard product and $\mathbf{h} = [h_1 \dots h_N]^T$, $\mathbf{g} = [g_1 \dots g_N]^T$.

Let $\bar{h}_i = E\{h_i\}$, $\alpha_i = E\{|h_i - \bar{h}_i|^2\}$, $\bar{g}_i = E\{g_i\}$, $\beta_i = E\{|g_i - \bar{g}_i|^2\}$ and assume that $\{h_i\}_{i=1}^N$, $\{g_i\}_{i=1}^N$, η_i and n_d are mutually independent, after some straightforward manipulations, the matrix $\mathbf{R}$ can be rewritten in terms of the means and the variances of the forward and the backward channels as follows

$$R_{i,j} = \begin{cases} P_s\left(|\bar{h}_i|^2 + \alpha_i\right)\left(|\bar{g}_i|^2 + \beta_i\right) & \text{if} \quad i = j \\ P_s \bar{h}_i \bar{g}_i \bar{h}_j^* \bar{g}_j^* & \forall i \ne j \end{cases} \tag{5}$$

The total noise power, denoted as P_n, is defined as

$$P_n = E\left\{\left(\sum_{i=1}^{N} w_i g_i \eta_i + n_d\right)\left(\sum_{j=1}^{N} w_j g_j \eta_j + n_d\right)^*\right\} \tag{6}$$

Assuming that η_i is a zero-mean additive noise and that $\{\eta_i\}_{i=1}^N$ are mutually independent random variables, then $E\{\eta_i \eta_j^*\} = 0 \quad \forall i \ne j$. Therefore, the noise power is given by

$$P_n = \mathbf{w}\mathbf{Q}\mathbf{w} + \sigma_d^2 \tag{7}$$

being $\mathbf{Q} = \sigma_r^2 diag(E\{|g_1|^2\}, E\{|g_2|^2\}, \dots, E\{|g_N|^2\})$.

Using (4) and (7), the SNR in (3) is obtained as

$$\text{SNR} = \frac{P_d}{P_n} = \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\mathbf{Q}\mathbf{w}+\sigma_d^2} \quad (8)$$

In order to formulate the power constraints at the relays in (3), the average transmit power of the ith relay, denoted as p_i, is defined as follows

$$p_i = E\left\{|x_i|^2\right\}|w_i|^2 = D_{ii}\,|w_i|^2 \quad (9)$$

being D_{ii} the (i,i)th entry of the diagonal matrix $\mathbf{D}$ which is given by $\mathbf{D} = P_s\ diag\left(E\left\{|h_1|^2\right\}, \ldots, E\{|h_N|^2\}\right) + \sigma_r^2\mathbf{I}$.

Considering (8) and (9), the maximization of SNR subject to per-relay constraints in (3) can be formally expressed as

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\mathbf{Q}\mathbf{w}+\sigma_d^2} \\ \text{s.t.} \quad & D_{ii}\,|w_i|^2 \le P_i \qquad \forall i = 1,...,N \\ & \text{card}(\mathbf{w}) = K \end{aligned} \quad (10)$$

This problem is hard combinatorial problem which requires an exhaustive search over all the possible $\binom{N}{K}$ sparsity patterns. This search is computationally unfeasible and this fact motivates the pursuit of an efficient algorithm with a near-optimal performance.

III. The proposed method

A. Selection of the subset

The problem presented in (10) is not convex. The classical approach in combinatorial optimization is to derive a convex relaxation of the original problem. Recently, in this context, the use of sparsity-inducing norms has been shown to be a powerful tool to obtain computationally tractable algorithms [9]. The most common approach to circumvent the computational bottleneck of combinatorial problems is to relax the original problem by replacing the cardinality constraint by the convex l_1-norm, defined as $\|\mathbf{w}\|_1 = \sum_{i=1}^{i=N}|w_i|$. Nevertheless, similar to [7], the l_1-norm squared, denoted as $\|\mathbf{w}\|_1^2$, is considered instead. The rationale behind the use of l_1-norm squared as a surrogate of the cardinality instead of the l_1-norm is twofold. First, it promotes the appearance of zeros in the network beamformer and, consequently, performs the multiple relay selection. Second, the relaxation of the problem (10) with the l_1-norm squared naturally yields a semidefinite programming (SDP). Something that is not obvious when the l_1-norm is considered instead. Let us relax the problem (10) by using the l_1-norm squared

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\mathbf{Q}\mathbf{w}+\sigma_d^2} \quad (11a)$$

$$\text{s.t.} \quad D_{ii}\,|w_i|^2 \le P_i \qquad \forall i = 1,...,N \quad (11b)$$

$$\|\mathbf{w}\|_1^2 \le \gamma \quad (11c)$$

being γ a positive parameter that controls the number of active elements in $\mathbf{w}$ and consequently, the number of selected relays. Let us skip at this point the discussion about how to adjust this parameter to carry out the selection of the best subset K cooperating relays. This issue will be addressed in the next subsection.

The problem (11) is still NP-hard and this motivates the use of the semidefinite relaxation (SDR) to handle it. Let us rewrite (11) in terms of $\mathbf{X} := \mathbf{w}\mathbf{w}^H$. With this aim in mind the constraint (11c) can be expressed as

$$\|\mathbf{w}\|_1^2 = \left(\sum_{i=1}^{i=N}|w_i|\right)^2 = \mathbf{1}_N^T\,|\mathbf{X}|\,\mathbf{1}_N \quad (12)$$

being $|\mathbf{X}|$ the element-wise absolute value of $\mathbf{X}$ and $\mathbf{1}_N$ a column-vector of length N. By substituting the l_1-norm squared by its equivalent formulation (12), the problem in (11) can be rewritten as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \frac{\text{Tr}\{\mathbf{R}\mathbf{X}\}}{\text{Tr}\{\mathbf{Q}\mathbf{X}\}+\sigma_d^2} \\ \text{s.t.} \quad & X_{ii} \le v_i\ \forall i = 1,...,N \\ & \mathbf{1}_N^T\,|\mathbf{X}|\,\mathbf{1}_N \le \gamma; \quad \mathbf{X} \succeq 0 \\ & \text{rank}\,(\mathbf{X}) = 1 \end{aligned} \quad (13)$$

being v_i the ith component of the vector $\mathbf{v}$ defined as $\mathbf{v} = [P_1/D_{11}, ..., P_N/D_{NN}]^T$ and X_{ii} the $(i,i)th$ entry of $\mathbf{X}$. By dropping the non-convex rank-one constraint the next problem is obtained

$$\max_{\mathbf{X}} \quad \frac{\text{Tr}\{\mathbf{R}\mathbf{X}\}}{\text{Tr}\{\mathbf{Q}\mathbf{X}\}+\sigma_d^2} \quad (14a)$$

$$\text{s.t.} \quad X_{ii} \le v_i \quad \forall i = 1,...,N \quad (14b)$$

$$\mathbf{1}_N^T\,|\mathbf{X}|\,\mathbf{1}_N \le \gamma; \quad \mathbf{X} \succeq 0 \quad (14c)$$

which is a quasi-convex problem in the variable $\mathbf{X}$. The standard approach for solving this type of problems in the signal processing literature is to use a bisection search method [2] in which the solution is sequentially searched by solving a sequence of (often many) semidefinite programming problems. Herein a simpler alternative for solving (14) is proposed. The idea is to reformulate the problem into a SDP using the Charnes-Cooper transformation. By using the following transformation of variables

$$z = \frac{1}{\text{Tr}\{\mathbf{Q}\mathbf{X}\}+\sigma_d^2}, \quad \mathbf{Y} = \frac{\mathbf{X}}{\text{Tr}\{\mathbf{Q}\mathbf{X}\}+\sigma_d^2} = z\mathbf{X} \quad (15)$$

the problem (14) is rewritten as

$$\max_{\mathbf{Y},\ z} \quad \text{Tr}\{\mathbf{R}\mathbf{Y}\} \quad (16a)$$

$$\text{s.t.} \quad Y_{ii} \le z\ v_i \quad \forall i = 1,...,N \quad (16b)$$

$$\mathbf{1}_N^T\,|\mathbf{Y}|\,\mathbf{1}_N \le z\gamma \quad (16c)$$

$$\text{Tr}\{\mathbf{Q}\mathbf{Y}\}+\sigma_d^2 z \ = \ 1 \quad (16d)$$

$$\mathbf{Y} \succeq 0; \ \ z > 0 \quad (16e)$$

which is a SDP in the variables $\mathbf{Y} \in H_N^+$ and $z \in \mathbb{R}_+$ and can be solved using standard interior point solvers. If $(\mathbf{Y}^*, z^*)$ is the optimal solution of (16), then $\mathbf{X}^* = \mathbf{Y}^*/z^*$ is the optimal solution to (14).

The subset of selected relays is determined as follows: the non-zero entries of the diagonal of $\mathbf{Y}^*$ correspond to relays selected for the cooperation. Contrarily, the null elements of the diagonal correspond to the relays that will not be considered for the retransmission.

B. Computation of the network beamformer weights

After the selection of the subset of relays, the weights of the selected relays need to be computed. It is worth noting that due to the influence of the l_1-norm squared behind the constraint (16c), the beamformer coefficients cannot be directly extracted from the solution of (16). To find the weights which maximize the SNR at the destination, we need to solve the reduced-size problem for the selected relays. With this aim in mind, the inactive relays and the constraint (16c) have to be removed from the problem. Let us denote by denote by $S \subseteq \{1, \ldots, N\}$ the subset of K nodes selected for the retransmission and by $\tilde{\mathbf{w}} = \left[w_{S(1)}, ..., w_{S(K)}\right]^T$ the weights of the active relays. To find the coefficients of the optimal beamforming the following reduced-size problem has to be solved:

$$\max_{\tilde{\mathbf{Y}},\, z} \quad \mathrm{Tr}\{\tilde{\mathbf{R}}\tilde{\mathbf{Y}}\} \tag{17a}$$

$$\text{s.t.} \quad \tilde{Y}_{ii} \leq z\, \tilde{v}_i \quad \forall i = 1, ..., K \tag{17b}$$

$$\mathrm{Tr}\{\tilde{\mathbf{Q}}\tilde{\mathbf{Y}}\} + \sigma_d^2 z \ = \ 1 \tag{17c}$$

$$\tilde{\mathbf{Y}} \succeq 0, \ \ z > 0 \tag{17d}$$

being $\tilde{\mathbf{R}}$ and $\tilde{\mathbf{Q}}$ submatrices of $\mathbf{R}$ and $\mathbf{Q}$ obtained after selecting the rows and columns corresponding to the active relays, $\tilde{v}_i$ denotes the ith entry of the vector $\tilde{\mathbf{v}}$ which is obtained after removing the inactive relays from the vector $\mathbf{v}$. In the same way, $\tilde{\mathbf{Y}}$ is a square matrix of size K formed by the active rows and columns of $\mathbf{Y}$.

Let us denote by $(\tilde{\mathbf{Y}}^*, z^*)$ the solution of (17). Due to the rank relaxation, $\tilde{\mathbf{Y}}^*$ may not be rank one, in general. For instances where $\tilde{\mathbf{Y}}^*$ happens to be rank one, the relaxation is tight and weights of the selected subset of relays $\tilde{\mathbf{w}}$ can be simply extracted from the eigendecomposition of the rank-one matrix $\mathbf{\Phi} = \tilde{\mathbf{Y}}^*/z^*$ (essentially, the principal eigevector of $\mathbf{\Phi}$). When the rank($\tilde{\mathbf{Y}}^*$)> 1, an approximate solution can be found using the randomization techniques exposed in [2].

C. Selection of the parameter γ

The proper choice of γ is of paramount importance because it controls the number of selected relays. It should be remarked that sparser solutions are obtained when γ is decreased. In this subsection we propose a method, based on binary search over γ, that successively increases the sparsity of the solution until the desired number of relays is selected. With this aim in mind, recall the relaxed problem (11) and the inequality (11c), i.e., $\|\mathbf{w}\|_1^2 \leq \gamma$, and consider the next useful bounds over the l_1-norm squared

$$\|\mathbf{w}\|_2^2 \leq \|\mathbf{w}\|_1^2 \leq K\, \|\mathbf{w}\|_2^2 \tag{18}$$

being $\|\mathbf{w}\|_2^2$ the square of the Euclidean norm. The expression (18) connects the l_1-norm squared with the desired cardinality K and the Euclidean norm. We need to determine an initial value γ in the binary search which guarantees that the final selection vector will have, at least, K active entries. This value will be denoted as $\gamma_{\max}$. Focusing on the right hand of the inequalities in (18), i.e., $\|\mathbf{w}\|_1^2 \leq K\, \|\mathbf{w}\|_2^2$, it is obvious that determining an upper bound on the Euclidean norm will be useful for the computing the value of $\gamma_{\max}$. To obtain this bound consider the problem in (17) assuming that all the relays are active, i.e., consider $\tilde{\mathbf{R}} = \mathbf{R}$ and $\tilde{\mathbf{Q}} = \mathbf{Q}$ and $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}$ and let $\mathbf{w}^{(0)}$ be the optimal beamformer obtained from the solution of this problem. From (11c) and (18), it is clear that $\gamma = K\left\|\mathbf{w}^{(0)}\right\|_2^2$ ensures that at least K relays will be active. This is due to the fact that by decreasing γ one is also decreasing $\|\mathbf{w}\|_1^2$ and, consequently, $\|\mathbf{w}\|_2^2$. Unfortunately, $\gamma = \gamma_{\max}$ often enforces solutions with more than K active entries in the solution vector. Therefore, we need to decrease the parameter γ by considering a binary search until a solution with the desired number of active relays is obtained. The whole algorithm is summarized in the following subsection. Note that this binary search requires solving (16) for different values of γ until a solution with the desired degree of sparsity is obtained. Nevertheless, the number of semidefinite programming problems which needs to be solved with this binary search is far less than in the exhaustive search which requires solving $\binom{N}{K}$ problems of type (17). This will be analyzed later with numerical results.

D. Description of the algorithm and analysis of the power reduction

The whole method is summarized in Algorithm 1.

Algorithm 1 Proposed method

STEP 1) INITIALIZATION: Solve (17) assuming that all the relays are active and obtain $\mathbf{w}^{(0)}$. Initialize the values for the binary search: $\gamma_{\max} = K\left\|\mathbf{w}^{(0)}\right\|_2^2$, $\gamma_{low} = 0$, $\gamma = \gamma_{\max}$.

STEP 2) SELECTION OF THE SUBSET OF RELAYS:
while number of active relays$\neq K$ **do**

A) Solve (16) for the corresponding γ and determine the active relays (non-zero elements of the diagonal of $\mathbf{Y}$).

B) Compute the new value of γ as follows

if number of active relays $> K$ **then**
$\gamma_{up} = \gamma$ and $\gamma \leftarrow (\gamma_{low} + \gamma)/2$
else
if number of active relays $< K$ **then**
$\gamma_{low} = \gamma$ and $\gamma \leftarrow (\gamma_{up} + \gamma)/2$
end if
end if
end while

STEP 3) COMPUTATION OF THE WEIGHTS: Solve the reduced-size problem (17) with the selected subset and extract the weights $\tilde{\mathbf{w}}$.

Power saving is a consequence of the proposed method. To analyze this fact let us explore the expression of the total power transmitted by the relays, which is given by

$$P_T = \sum_{i=1}^{N} |y_i|^2 = \sum_{i=1}^{N} E\left\{|x_i|^2\right\}|w_i|^2 = \mathbf{w}^H\mathbf{D}\mathbf{w} \tag{19}$$

where y_i and the diagonal matrix $\mathbf{D}$ have been defined in Subsection II-A. Applying the Cauchy-Schwarz inequality we obtain $P_T = \mathbf{w}^H\mathbf{D}\mathbf{w} = \left\|\mathbf{D}^{1/2}\mathbf{w}\right\|_2^2 \leq \left\|\mathbf{D}^{1/2}\right\|_2^2 \|\mathbf{w}\|_2^2$. Thus, taking into account the left side inequality in (18) and (11c), it is straightforward to show

$$P_T \leq \left\|\mathbf{D}^{1/2}\right\|_2^2 \|\mathbf{w}\|_2^2 \leq \left\|\mathbf{D}^{1/2}\right\|_2^2 \|\mathbf{w}\|_1^2 \leq \left\|\mathbf{D}^{1/2}\right\|_2^2 \gamma \quad (20)$$

Note that by decreasing γ to promote the desired degree of sparsity, the power transmitted by the relays is also decreased.

IV. Simulation results

In this section numerical results are presented to show the performance of the proposed algorithm. To solve the SDP problems CVX [10] is used. The considered scenario is composed of a source which transmits with power $P_s = 0$ dBW, a destination and $N = 11$ potential relays whose individual power constraints are uniformly given by $P_i = 0$ dBW. The means of the forward and the backward channels are generated randomly as $\bar{h}_i, \bar{g}_i \sim \mathcal{CN}(0,1)$ and the variances are generated as $\alpha_i, \beta_i \sim \frac{1}{2}\mathcal{X}^2(2)$, where $\mathcal{X}^2(2)$ denotes the chi-square distribution with two degrees of freedom. The noise variances are set to $\sigma_d^2 = \sigma_r^2 = -3$ dBW. Bearing in mind this set of values, $\mathbf{R}$, $\mathbf{Q}$ and $\mathbf{D}$ are generated following the expressions presented in the Subsection II-A, taking into account that $E\{|h_i|^2\} = |\bar{h}_i|^2 + \alpha_i$ and $E\{|g_i|^2\} = |\bar{g}_i|^2 + \beta_i$.

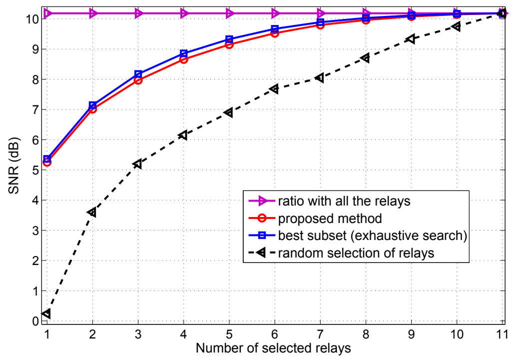

Fig. 2. SNR as a function of the number of selected relays K.

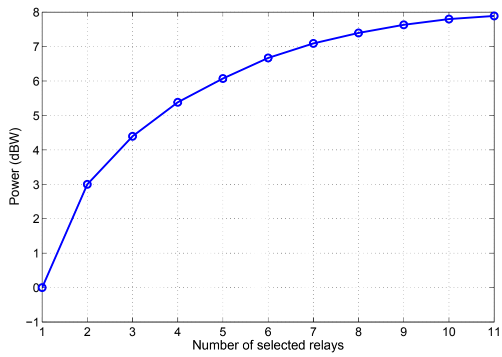

Fig. 3. Power transmitted by the relays versus the number of selected relays.

Fig. 2 and Fig. 3 show the SNR and the total power transmitted by the relays as a function of the number of selected relays K, respectively. The curves were obtained by averaging 60 simulation runs. In each trial, $\mathbf{R}$, $\mathbf{Q}$ and $\mathbf{D}$ have been generated according to the procedure described above. Note that in Fig. 2 the performance of the proposed algorithm is very close to the one of the exhaustive search. Furthermore, the proposed method has a computational burden that is far less than the one required by the exhaustive search. The mean number of SDP problems that needs to be solved in the proposed technique for any value of K is less than 7 (less than 6 for the selection of the subset plus one for the computation of the weights) and the worst-case, which was obtained for $K = 6$, has required the computation of 13 SDP. This value is far less than the number of SDP needed by the exhaustive search which requires solving $\binom{11}{6} = 462$ SDP.

From the analysis of Fig. 2 and Fig. 3, we can conclude that by selecting the best subset of 2 relays instead of using all nodes, almost 5 dB of transmitted power can be saved at expenses of around 3 dB of SNR performance.

V. Conclusions

A new method has been proposed for the selection of the best subset of relays, in a distributed beamforming scheme, which maximizes the SNR at the destination subject to per-relay constraints. Whereas this is a problem of combinatorial nature which requires an exhaustive search, the proposed method achieves a performance that is very close to the SNR-optimal multiple relay selection with an affordable computational burden. Our method assumes the knowledge of the second-order statistics of channels and the relays are not limited to cooperate at full power.

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