

Compressed Correlation-Matching for Spectrum Sensing in Sparse Wideband Regimes

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Abstract—In this paper, we consider a novel compressed correlation-matching (CCM) approach for spectrum sensing of wideband sparse signals. We derive a general closed-form estimate of the wideband sparse signal level from compressed observations, while providing physical interpretation of the problem. The formulation allows straightforward application to signal processing problems of interest, such as generalized likelihood ratio test (GLRT) spectrum sensing for wideband cognitive radio. Simulation results are reported to assess the behavior of the CCM method.

Index Terms—Correlation-matching, compressed-sampling, wideband regime, cognitive radio, spectrum sensing.

I. INTRODUCTION

Maximum likelihood (ML) estimation is an important method used in a wide range of statistical signal processing problems [1]. However, despite its theoretical asymptotic optimality, ML is often difficult to implement, and analytical solutions to the maximization problem are available only in a few circumstances [2]. In wideband regimes, ML becomes further challenging because wideband signals are characterized by close to zero spectral efficiency and very low signal-to-noise ratios (SNRs) [3]. As an alternative, we propose the application of the correlation-matching technique to spectrum sensing of wideband signals. Correlation-matching is a least-squares fitting of second-order statistics, and behaves as an approximation to ML at the low-SNR regime with asymptotic large data records [4].

Due to the low occupancy of many communication systems, we recognize that primary signals are sparse in the spectrum domain, which allows the application of the foreseen theory of compressed-sampling [5] to further reduce the sampling rate required for spectrum sensing detection. Conventional compressed-sampling is based on heuristic algorithms, e.g., basis pursuit [5], and hereby lack of physical interpretation. A comparison addressed by [6] shows that nonuniform sampling does not suffer from some drawbacks present in traditional uniform (Nyquist) sampling.

Spectrum sensing is a primary function of interweave cognitive radio [7], and consists of reliably identifying the available spectrum resources temporally unused by the primary

users [8]. The performance of the most commonly employed spectrum sensing techniques, such as the energy detector [9], pilot-based methods [10], cyclostationarity detection [11] and match-filtering [12], is severely degraded with inaccurate prior information on the model features. In this work, we consider the generalized likelihood ratio test (GLRT) [13] for spectrum sensing in wideband cognitive radio [7]. GLRT has received recent attention [14]–[16] because it is optimal in the Neyman-Pearson sense, and it incorporates ML estimation for inaccurate model parameters.

In this work, we discuss the compressed-sampling version of the correlation-matching approach for spectrum reconstruction of sparse wideband signals. The potentials of correlation-matching as a matrix-level fitting has been recently addressed by [17], and we reformulate it under the compressed-sampling scenario for asymptotically sparse wideband regimes. We derive a general closed-form estimate of the signal power level based on compressed observations. This unified formulation allows physical interpretation of the problem, and straightforward application to the problem of GLRT spectrum sensing in multi-frequency cognitive radios.

II. SIGNAL DESCRIPTIONS AND PROBLEM STATEMENT

We consider the compressed correlation-matching (CCM) problem on the second-order statistics of a wideband signal $S(t)$, which may represent, but not limited to, the superposition of the primary services in a cognitive radio network. The sensed signal is of the form $X(t) = S(t) + W(t)$, where $W(t)$ is the double-sided complex zero-mean additive white Gaussian noise with spectral density $N_0/2$. The m -th discrete-time signal is defined as $\mathbf{x}_m \doteq [X(t_1^m), \dots, X(t_N^m)]^T$, $1 \leq m \leq M$, where N is the observation size and M the sampling depth. The intervals satisfy Nyquist-rate uniform sampling and piece-wise stacking, i.e., $t_n^m = (mN + n)\Delta_x$, where $\Delta_x = \frac{1}{B}$. In this formulation, the sensing is performed over the frequency range $f \in [0, B)$. As a result, the observations consists of a data record of size $N \times M$ given by $\mathbf{X} \doteq (\mathbf{x}_1, \dots, \mathbf{x}_M)$. We similarly define \mathbf{S} and \mathbf{W} for the signal and noise components of the model. In wideband regimes, as $B \rightarrow \infty$, an impractical large number of samples would be required. However, we recognize that the relative occupation of $S(t)$ decreases as the sensed bandwidth

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increases, which motivates the application of the compressed-sampling theory [5] to further reduce the sampling rate to sense $S(t)$. Let $\phi_s(f)$ be the spectrum of $S(t)$. In the sequel, we state that $S(t)$ has a sparse spectrum if the support of $\phi_s(f)$ is small compared to the sensed bandwidth, i.e., if $\mathcal{L}(\mathcal{P}_s) \ll B$, where $\mathcal{L}(\mathcal{P}_s)$ denotes the Lebesgue measure of the frequency set $\mathcal{P}_s \doteq \{f \in [0, B) : \phi_s(f) > 0\}$. In other words, we investigate the sensing of $S(t)$ from a smaller K -dimensional discrete-time signal \mathbf{y}_m linearly related to \mathbf{x}_m through the $K \times N$ compression matrices Ψ_m , i.e.,

$$\mathbf{y}_m = \Psi_m \mathbf{x}_m = \Psi_m (\mathbf{s}_m + \mathbf{w}_m). \quad (1)$$

with $K < N$. In (1), Ψ_m is a pinning matrix that randomly selects K samples of \mathbf{x}_m , and it is given by randomly selecting K rows of \mathbf{I}_N . In average, the sampling rates of \mathbf{y}_m and \mathbf{x}_m are related through the compression rate, defined as $\kappa \doteq \frac{K}{N}$, by $\frac{1}{\Delta_y} = \kappa \frac{1}{\Delta_x}$. We further define the occupation rate as the ratio $\kappa_0 \doteq \frac{\mathcal{L}(\mathcal{P}_s)}{B}$. As the spectral information of $S(t)$ is contained in the second-order statistics, we propose a CCM approach based on nonuniform sub-Nyquist rate sampling, and its application to spectrum sensing in sparse wideband cognitive radio.

III. COMPRESSED CORRELATION-MATCHING APPROACH

The reconstruction of the second-order statistics of $X(t)$ based on the compressed observations $\mathbf{Y} \doteq (\mathbf{y}_1, \dots, \mathbf{y}_M)$ is next discussed. Let Θ denote the set of parameters that uniquely characterize the second-order statistics of the received signal. The CCM approach is then based on a second-order fitting between the observations and the correlation model $\mathcal{R}(\Theta)$.

Definition 1. Let $\mathcal{R}(\Theta)$ be the parameterized second-order statistics of $X(t)$. The CCM metric \mathcal{M} is defined as $\mathcal{M}[\mathbf{Y}, \mathcal{R}(\Theta)] \doteq \|\mathbf{Y}\mathbf{Y}^H - \sum_m \Psi_m \mathcal{R}(\Theta) \Psi_m^H\|^2$, and the approach for recovering Θ is given by

$$\hat{\Theta} = \arg \min_{\Theta} \mathcal{M}[\mathbf{Y}, \mathcal{R}(\Theta)]. \quad (2)$$

The advantage of the formulation (2) is that it generalizes the CCM approach for any correlation model $\mathcal{R}(\Theta)$, which permits further extensions to spectrum sensing detection. In many practical situations, the prior knowledge on noise and signal statistics of (1) enables signal processing algorithms for both detection and estimation problems. Moreover, because in compressed-sampling the received observations are projected onto a lower dimensional space, the phenomenon of noise enhancement is observed at the output of the reconstruction. For the purpose of diminishing this effect, the correlation model $\mathcal{R}(\Theta)$ is cast as

$$\mathcal{R}(\gamma, \sigma^2) = \gamma \mathbf{R}_0 + \sigma^2 \mathbf{I}_N, \quad (3)$$

where \mathbf{R}_0 denotes the normalized signal correlation matrix, e.g., with $\text{tr}(\mathbf{R}_0) = N$, and γ and σ^2 denote the signal and noise power levels over B , respectively. We further define the nominal SNR as $\rho_0 \doteq \frac{\gamma}{\sigma^2}$. In the problem at hand, we assume that both noise and signal terms are complex zero-mean Gaussian distributed. While facilitating the analysis,

it is a reasonable assumption because usually there is no line-of-sight (LOS) path between the sparse wideband signal source and the receive antenna. Hence, the resulting signal is the superposition of no-LOS signals and approximates to the Gaussian distribution as the number of observations is sufficiently large, according to the central limit theorem. In a cognitive radio environment, the signal detection performance is fostered by the prior knowledge on the primary systems' normalized statistics, e.g., the modulation formats employed by the primary services. In what follows, we state the main result of our paper.

Theorem 1. Consider the correlation model (3) of $X(t)$. The CCM estimate (CCME) of the signal contribution that minimizes $\mathcal{M}[\mathbf{Y}, \mathcal{R}(\gamma, \sigma^2)]$ is given by

$$\hat{\gamma} = \frac{\text{tr}[\mathbf{R}_y (\bar{\mathbf{R}}_0 - \frac{1}{K} \text{tr}(\bar{\mathbf{R}}_0) \mathbf{I}_K)]^+}{\text{tr}(\bar{\mathbf{R}}_0^2) - \frac{1}{K} \text{tr}^2(\bar{\mathbf{R}}_0)}, \quad (4)$$

where $\mathbf{R}_y \doteq \frac{1}{M} \mathbf{Y}\mathbf{Y}^H$ is the sample covariance matrix of the compressed observations, $\bar{\mathbf{R}}_0 \doteq \frac{1}{M} \sum_{m=1}^M \Psi_m \mathbf{R}_0 \Psi_m^H$ is the compressed version of the normalized signal correlation matrix, and $x^+ \doteq \max(0, x)$.

Proof: See Appendix A. ■

The physical interpretation of the CCME (4) can be discussed from the compressed statistics \mathbf{R}_y , and $\bar{\mathbf{R}}_0$. It can be appreciated that the expression of the numerator of $\hat{\gamma}$ can be rewritten as the scalar product $\text{tr}(\mathbf{R}_y \mathbf{R}_D)$, where we have defined the $K \times K$ diagonal off-loaded signal correlation matrix as $\mathbf{R}_D \doteq \bar{\mathbf{R}}_0 - \frac{1}{K} \text{tr}(\bar{\mathbf{R}}_0) \mathbf{I}_K$. The first term of \mathbf{R}_D performs the projection of the sample covariance matrix \mathbf{R}_y onto the compressed normalized signal space defined by $\bar{\mathbf{R}}_0$. The second term is in charge of subtracting the part of the reconstructed observations that are considered as noise according to the signal model (1). Because the spectrum of $S(t)$ is strongly sparse, the average energy is measured in the compressed domain, i.e., over $\bar{\mathbf{R}}_0$, rather than directly on \mathbf{R}_0 . If $S(t)$ is a stationary process, it is noticed that $\mathbf{R}_D = \bar{\mathbf{R}}_0 - \mathbf{I}_K$, because $\bar{\mathbf{R}}_0$ has an all-ones main diagonal. The projection of \mathbf{B} is then performed onto the components of $\bar{\mathbf{R}}_0$ that are not affected by the noise, i.e., the non-zero correlation lags contained out of the main diagonal. Conversely, if $S(t)$ is cyclostationary, the main diagonal of $\bar{\mathbf{R}}_0$ is not uniform, and \mathbf{R}_D evaluates the energetic variability around the average energy, in addition to the correlation of the non-zero lags.

Moreover, the denominator of (4) reflects the distinctness between $\bar{\mathbf{R}}_0$ and $\bar{\mathbf{I}}_N$, as it becomes subjected to the Cauchy-Schwarz inequality. Let $\langle \bar{\mathbf{A}}, \bar{\mathbf{B}} \rangle$ be the inner product between $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$, defined as $\langle \bar{\mathbf{A}}, \bar{\mathbf{B}} \rangle \doteq \text{tr}[(\frac{1}{M} \sum_m \Psi_m \bar{\mathbf{A}} \Psi_m^H) (\frac{1}{M} \sum_{m'} \Psi_{m'} \bar{\mathbf{B}} \Psi_{m'}^H)]$. Further consider the squared norm of $\bar{\mathbf{A}}$ as $\|\bar{\mathbf{A}}\|^2 \doteq \langle \bar{\mathbf{A}}, \bar{\mathbf{A}} \rangle$. By writing K times the denominator of (4), we may rewrite each term in function of inner products and norms, i.e., $K \text{tr}(\bar{\mathbf{R}}_0^2) = \|\bar{\mathbf{R}}_0\|^2 \|\bar{\mathbf{I}}_N\|^2$, and $\text{tr}^2(\bar{\mathbf{R}}_0) = \langle \bar{\mathbf{R}}_0, \bar{\mathbf{I}}_N \rangle^2$. After taking the squared root of both terms, we show the non-negativity of

the denominator, because it holds that

$$\|\bar{\mathbf{R}}_0^2\| \|\bar{\mathbf{I}}_N^2\| \geq \langle \bar{\mathbf{R}}_0^2, \bar{\mathbf{I}}_N^2 \rangle. \quad (5)$$

It is noticed that the more distinct are the second-order statistics of the signal and the noise, the more robust is the estimate (4). The equality in (5) only holds when the signal and noise statistics are linearly dependent, i.e., both white, or when $K = 1$, for which in the sequel we assume $K > 1$.

IV. APPLICATION TO MULTI-FREQUENCY SPECTRUM SENSING

We recognize that wideband cognitive radio networks are characterized by a sparse primary systems' spectrum, which motivates the application of the CCM scheme to spectrum sensing. We now propose $S(t)$ in (1) as a multi-frequency system given by $S(t) \doteq \sum_{l=1}^L S_l(t)$, i.e., the addition of primary signals all with the same spectral pattern and occupying L adjacent bands. This model actually corresponds to many real licensed systems such as the terrestrial digital video broadcasting (DVB-T) standard for digital television. The correlation model $\mathcal{R}(\Theta)$ for multi-frequency systems is hereby given by

$$\mathcal{R}(\gamma_1, \dots, \gamma_L, \sigma^2) \doteq \sum_{l=1}^L \gamma_l \mathbf{R}_l + \sigma^2 \mathbf{I}_N, \quad (6)$$

where $\mathcal{W} \doteq \{\mathbf{R}_l\}$ is the set of normalized correlation matrices of uniform Nyquist-rate sampled versions of $S_l(t)$, with $\text{tr}(\mathbf{R}_l) = N$, and γ_l denote the power levels. ML estimation, which constitutes a natural subprocess within the spectrum sensing under the GLRT perspective, offers robustness in front of inaccurate model parameters. In the sequel, we discuss the CCM approach as an alternative to ML in the low-SNR regime, and its application to spectrum sensing.

A. Compressed Correlation-Matching Estimates

For given \mathcal{W} , the compressed GLRT spectrum sensing of multi-frequency systems involves the estimation of the signal and noise power levels from the compressed observations \mathbf{Y} [13]. The following Theorem generalizes the CCME (4) to multi-frequency signals.

Theorem 2. Consider the multi-frequency model (6). The solution of the signal and noise contributions under the CCM $\mathcal{M}[\mathbf{Y}, \mathcal{R}(\gamma_1, \dots, \gamma_L, \sigma^2)]$ is given by the solution of the system of equations

$$\begin{pmatrix} \text{tr}(\bar{\mathbf{R}}_1^2) & \dots & \text{tr}(\bar{\mathbf{R}}_1 \bar{\mathbf{R}}_L) & \text{tr}(\bar{\mathbf{R}}_1) \\ \vdots & \ddots & \vdots & \vdots \\ \text{tr}(\bar{\mathbf{R}}_L \bar{\mathbf{R}}_1) & \dots & \text{tr}(\bar{\mathbf{R}}_L^2) & \text{tr}(\bar{\mathbf{R}}_L) \\ \text{tr}(\bar{\mathbf{R}}_1) & \dots & \text{tr}(\bar{\mathbf{R}}_L) & K \end{pmatrix} \times \begin{pmatrix} \hat{\gamma}_1 \\ \vdots \\ \hat{\gamma}_L \\ \hat{\sigma}^2 \end{pmatrix} = \begin{pmatrix} \text{tr}(\mathbf{R}_y \bar{\mathbf{R}}_1) \\ \vdots \\ \text{tr}(\mathbf{R}_y \bar{\mathbf{R}}_L) \\ \text{tr}(\mathbf{R}_y) \end{pmatrix}, \quad (7)$$

where $\mathbf{R}_y = \frac{1}{M} \mathbf{Y} \mathbf{Y}^H$ is the sample covariance matrix of the compressed observations, and $\bar{\mathbf{R}}_l \doteq \frac{1}{M} \sum_{m=1}^M \Psi_m \mathbf{R}_l \Psi_m^H$ is

the compressed version of the normalized signal correlation matrix at the l -th band.

Proof: See Appendix B. ■

The detection of the signal levels $(\gamma_1, \dots, \gamma_L)$ is based on the compressed observations \mathbf{Y} , which contain spectral information of the sensed bandwidth. Contrarily to conventional filter-bank techniques, the CCM approach is based on the system matrix (7) which reflects the cross-correlation between different bands given that orthogonality is not preserved after compressed-sampling.

B. Compressed Multi-Frequency Spectrum Sensing

As a joint multiple-hypotheses test, the complexity of the GLRT in multi-frequency systems grows exponentially with the number of bands, which becomes impractical due to the limited capabilities of the wideband cognitive radios. Conversely, the spectrum sensing problem at the l -th band may be cast as a binary hypothesis testing problem by treating the remaining frequencies as nuisance parameters [13], i.e.,

$$\begin{aligned} \mathcal{H}_{0,l} &: \mathbf{y}_m = \Psi_m \left(\sum_{k \neq l} \mathbf{s}_{m,k} + \mathbf{w}_m \right) \\ \mathcal{H}_{1,l} &: \mathbf{y}_m = \Psi_m \left(\mathbf{s}_{m,l} + \sum_{k \neq l} \mathbf{s}_{m,k} + \mathbf{w}_m \right), \end{aligned} \quad (8)$$

where $\mathbf{s}_{m,l} \doteq [S_l(t_1^m), \dots, S_l(t_N^m)]^T$. The complexity is then reduced to L binary tests. The next theorem summarizes the expression of the optimal spectrum sensing detector employing CCM.

Theorem 3. The optimal multi-frequency GLRT (MF-GLRT) for asymptotic wideband regime is given by

$$T_l(\mathbf{Y}|\mathcal{W}) = \text{tr} \left[\Xi_l^{-1} \hat{\gamma}_l \bar{\mathbf{R}}_l (\Xi_l + \hat{\gamma}_l \bar{\mathbf{R}}_l)^{-1} \mathbf{R}_y \right] \geq \lambda_l, \quad (9)$$

where $\Xi_l \doteq \sum_{k \neq l} \hat{\gamma}_k \bar{\mathbf{R}}_k + \hat{\sigma}^2 \mathbf{I}_K$ represents the equivalent noise-plus-interferences covariance when sensing the l -th band, $(\hat{\gamma}_1, \dots, \hat{\gamma}_L, \hat{\sigma}^2)$ are the CCME of the multi-frequency signal and noise levels, given by (7), and $\mathbf{R}_y \doteq \frac{1}{M} \mathbf{Y} \mathbf{Y}^H$ is the sample covariance matrix of the compressed observations.

Proof: Omitted due to length limitation. ■

In (9), the threshold λ_l is set to satisfy the false alarm constraint $\mathbb{P}[T_l(\mathbf{Y}|\mathcal{W}) \geq \lambda_l | \mathcal{H}_0] = \alpha_l$, which, without loss of generality, we set to $\alpha_l = \alpha$ for $l = 1, \dots, L$. While according to the GLRT theory $(\hat{\gamma}_1, \dots, \hat{\gamma}_L, \hat{\sigma}^2)$ are the ML estimates of the signal and noise power levels, we alternatively consider the CCME (7) as a valid approximation to ML in the low-SNR regime [4].

We see that the MF-GLRT (9) takes advantage of the occupation on the remaining frequencies as equivalent noise when sensing the l -th band. Therefore, the performance of (9) is affected by the signal-to-interference-plus-noise ratio (SINR) based on the cross-correlation that arises from the adjacent bands, and the signal and noise power level reconstruction in (7).

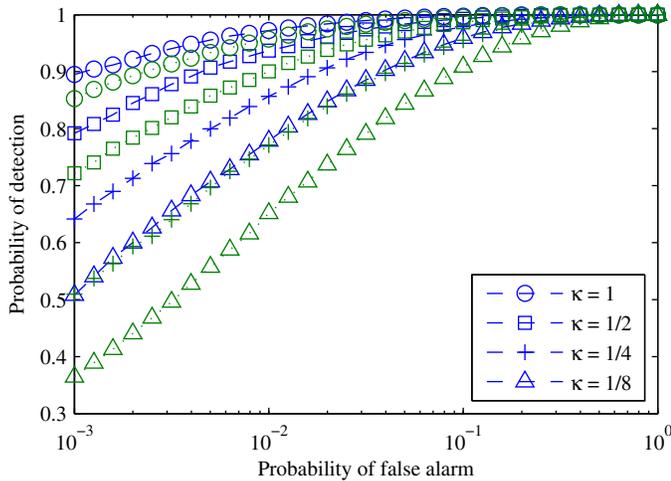


Fig. 1. Receiver operating characteristics (ROC) of the EC (dashed blue lines) and the MF-GLRT (9) (dotted green lines) in a wideband scenario with occupancy rate $\kappa_0 = \frac{1}{8}$ and SNR $\rho_0 = -12.5$ dB.

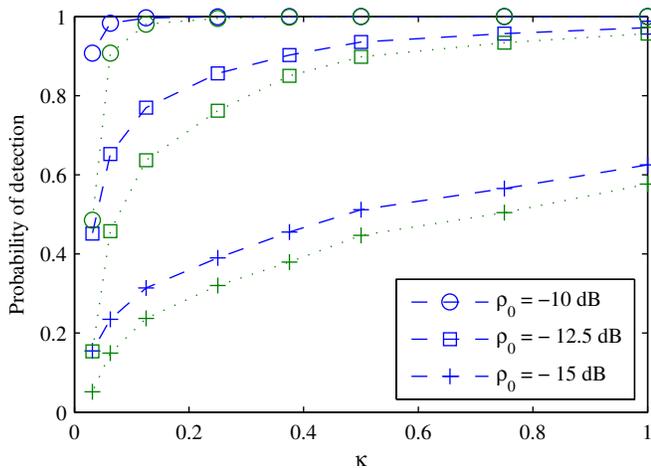


Fig. 2. Probability of detection performance of the EC (dashed blue lines) and the MF-GLRT (9) (dotted green lines) versus compression rate κ , with occupancy rate $\kappa_0 = \frac{1}{8}$ and false alarm level $\alpha = 0.01$.

V. NUMERICAL RESULTS

We finally provide simulation results to support the novel CCM approach in terms of spectrum sensing detection in multi-frequency cognitive radio. Specifically, we consider a cognitive radio network with primary systems based on the DVB-T standard in the 2k-mode with $L = 16$ bands, when sensing an arbitrary band. The size of the observation before compression is set to $N = 32$ samples. In order to strictly focus on the performance behavior due to compression and remove the effect of insufficient data records, we set $M(\kappa) = 2N\kappa^{-1}$, so that the size of the compressed observations database \mathbf{Y} is $2N$ for any compression rate. In other words, for high compression level (small κ), the cognitive radio takes samples from a larger period of time to preserve the total number of available samples. The average occupation of the system is $\kappa_0 = \frac{1}{8}$. For comparison purposes, we also

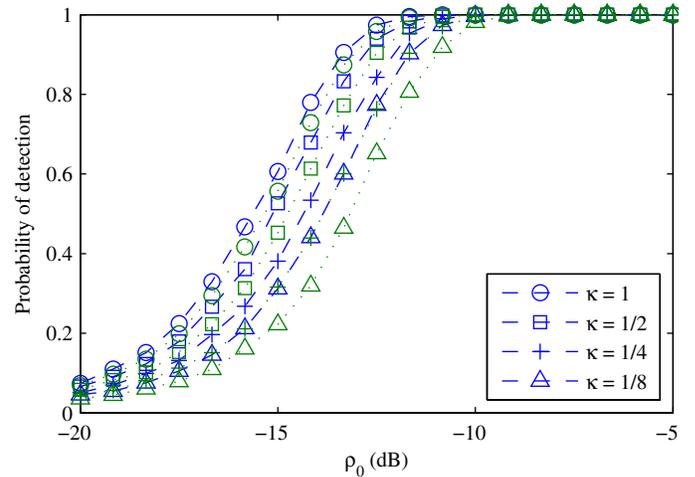


Fig. 3. Probability of detection performance of the EC (dashed blue lines) and the MF-GLRT (9) (dotted green lines) versus average SNR, with occupancy rate $\kappa_0 = \frac{1}{8}$, and false alarm level $\alpha = 0.01$.

include the estimator-correlator (EC) detector [13], i.e., the test statistic (9) with perfectly known signal and noise power levels $(\gamma_1, \dots, \gamma_L, \sigma^2)$.

Firstly, the receiver operating characteristics (ROC) of the MF-GLRT (9) and the EC are presented in Fig. 1 for several compression rates at an average SNR of $\rho_0 = -12.5$ dB. It is observed that the degradation in probability of detection incurred by the CCME of the signal and noise power levels (7) is small when compared to the performance bound established by the EC, for a wide range of false alarm levels. As the compression level increases (smaller κ), the ROC curves of both detectors suffer from degradation in terms of probability of detection, mainly for very restrictive false alarm levels.

Secondly, Fig. 2 plots the behavior of the MF-GLRT (9) and the EC versus the compression rate κ , in terms of probability of detection for several average SNR values. It is deduced that for a fixed probability of detection target, higher compression levels (smaller κ) are allowed as the average SNR increases. As an example, for an average SNR of $\rho_0 \geq -10$ dB, it is argued that for a false alarm level of $\alpha = 0.01$, the MF-GLRT offers outstanding probability of detection for a wide range of compression levels, i.e., it provides probabilities of detection near to $P_d = 1$ for $\kappa \geq \kappa_0$.

Finally, Fig. 3 depicts the probability of detection versus average SNR curves of the MF-GLRT (9) and the EC for several compression rates, for a false alarm level of $\alpha = 0.01$. While supporting the former interpretations, it can be appreciated from Fig. 1 and Fig. 3 that the performance loss incurred by the CCME (7) increases with higher compression rates (smaller κ), as the coupling between adjacent bands of the system matrix in (7) is larger as κ becomes smaller. Moreover, it is seen that the performance loss incurred by the MF-GLRT (9) is at most 1 dB in SNR when compared to the EC, even for a compression rate of the order of the primary systems occupation, i.e., for $\kappa = \kappa_0$.

VI. CONCLUSIONS

In this paper, we have investigated a novel CCM approach and its application to spectrum sensing of wideband sparse signals. The closed-form expression of the signal power level estimate has provided physical insight on the problem. The optimal MF-GLRT based on the CCME of multi-frequency signal and noise power levels has been derived. Numerical results have assessed the performance of the proposed technique, which remains tight to the EC bound for a wide range of compression rates, SNRs, and false alarm levels.

APPENDIX A PROOF OF THEOREM 1

Consider the correlation model $\mathcal{R}(\gamma, \sigma^2) \doteq \gamma \mathbf{R}_0 + \sigma^2 \mathbf{I}_N$. The signal and noise power levels that minimize the CCM (2) with compressed observations \mathbf{Y} are the solution of the optimization problem

$$(\hat{\gamma}, \hat{\sigma}^2) = \arg \min_{\gamma, \sigma^2} \mathcal{M}[\mathbf{Y}, \mathcal{R}(\gamma, \sigma^2)].$$

Noting the convexity of the problem, we take the derivative with respect to the signal level and set it to zero, which leads to the equation

$$\text{tr} \left[\left(\mathbf{Y} \mathbf{Y}^H - \sum_m \Psi_m (\gamma \mathbf{R}_0 + \sigma^2 \mathbf{I}_N) \Psi_m^H \right) \sum_m \Psi_m \mathbf{R}_0 \Psi_m^H \right] = 0.$$

Similarly, after taking the derivative with respect to the noise power level and equal it to zero, we obtain that

$$\text{tr} \left[\left(\mathbf{Y} \mathbf{Y}^H - \sum_m \Psi_m (\gamma \mathbf{R}_0 + \sigma^2 \mathbf{I}_N) \Psi_m^H \right) \sum_m \Psi_m \mathbf{I}_N \Psi_m^H \right] = 0.$$

Clearly, the derivatives outline the coupling between the signal and noise statistics, which is based on the projection of the local observations onto the normalized correlation matrices. By dividing each term in both equations by M , we define the following matrices $\mathbf{R}_y \doteq \frac{1}{M} \mathbf{Y} \mathbf{Y}^H$ and $\bar{\mathbf{R}}_0 \doteq \frac{1}{M} \sum_m \Psi_m \mathbf{R}_0 \Psi_m^H$. The derivatives can be then written as $\text{tr}(\mathbf{R}_y \mathbf{R}_0) - \gamma \text{tr}(\bar{\mathbf{R}}_0^2) - \sigma^2 \text{tr}(\bar{\mathbf{R}}_0) = 0$, and $\text{tr}(\mathbf{R}_y) - \gamma \text{tr}(\bar{\mathbf{R}}_0) - \sigma^2 K = 0$, respectively, where we made use of the property $\Psi_m \Psi_m^H = \mathbf{I}_K$. After some mathematical manipulations, it can be shown that the system of equations formed by the former derivatives has as solution (4).

APPENDIX B PROOF OF THEOREM 2

Consider the multi-frequency correlation model given by $\mathcal{R}(\gamma_1, \dots, \gamma_L, \sigma^2) \doteq \sum_{l=1}^L \gamma_l \mathbf{R}_l + \sigma^2 \mathbf{I}_N$. The multi-frequency signal and noise power levels that minimize the CCM (2) with compressed observations \mathbf{Y} are the solution to the optimization problem

$$(\hat{\gamma}_1, \dots, \hat{\gamma}_L, \hat{\sigma}^2) = \arg \min_{\gamma_1, \dots, \gamma_L, \sigma^2} \mathcal{M}[\mathbf{Y}, \mathcal{R}(\gamma_1, \dots, \gamma_L, \sigma^2)].$$

It is noticed that the problem is convex on the multi-frequency signal power levels, we take the derivative of

$\mathcal{M}[\mathbf{Y}, \mathcal{R}(\gamma_1, \dots, \gamma_L, \sigma^2)]$ with respect to the signal level at the l -th band and set it to zero, which leads to the equation

$$\text{tr} \left[\left(\mathbf{Y} \mathbf{Y}^H - \sum_m \Psi_m \left(\sum_k \gamma_k \mathbf{R}_k + \sigma^2 \mathbf{I}_N \right) \Psi_m^H \right) \sum_m \Psi_m \mathbf{R}_l \Psi_m^H \right] = 0.$$

Similarly, we take the derivative with respect to the noise power level and equal it to zero, which leads to the equation

$$\text{tr} \left[\left(\mathbf{Y} \mathbf{Y}^H - \sum_m \Psi_m \left(\sum_k \gamma_k \mathbf{R}_k + \sigma^2 \mathbf{I}_N \right) \Psi_m^H \right) \sum_m \Psi_m \mathbf{I}_N \Psi_m^H \right] = 0.$$

We note that now the derivatives also outline the coupling among the statistics of the remaining bands and the noise. By dividing each term of both equations by M and further defining the matrices $\mathbf{R}_y \doteq \frac{1}{M} \mathbf{Y} \mathbf{Y}^H$ and $\bar{\mathbf{R}}_l \doteq \frac{1}{M} \sum_m \Psi_m \mathbf{R}_l \Psi_m^H$ for $1 \leq l \leq L$, the derivatives can be written as $\text{tr}(\mathbf{R}_y \mathbf{R}_l) - \sum_k \gamma_k \text{tr}(\bar{\mathbf{R}}_k \bar{\mathbf{R}}_l) - \sigma^2 \text{tr}(\bar{\mathbf{R}}_l) = 0$, and $\text{tr}(\mathbf{R}_y) - \sum_{k=1}^L \gamma_k \text{tr}(\bar{\mathbf{R}}_k) - \sigma^2 K = 0$, respectively, where we made use of the property $\Psi_m \Psi_m^H = \mathbf{I}_K$. After some mathematical manipulations, we write the $L + 1$ equations in a system of equations, which proves (7).

REFERENCES

- [1] S. M. Kay, *Fundamentals of Statistical Signal Processing*. Upper Saddle River, NJ: Prentice Hall, 1998, vol. 1 (Estimation Theory).
- [2] B. Porat, *Digital Processing of Random Signals. Theory and Methods*. Dover Publications, Feb. 2008.
- [3] S. Verdu, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1319–1343, Jun. 2002.
- [4] W. A. Gardner, *Statistical Spectral Analysis. A Non Probabilistic Theory*. Prentice-Hall, 1988.
- [5] E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," *IEEE Signal Process. Mag.*, pp. 21–30, Mar. 2008.
- [6] P. Stoica, J. Li, and H. He, "Spectral analysis of nonuniformly sampled data: A new approach versus the periodogram," *IEEE Trans. Signal Process.*, vol. 57, no. 3, pp. 843–858, Mar. 2009.
- [7] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. of the IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [8] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 32–39, Apr. 2008.
- [9] H. V. Poor, *An Introduction to Signal Detection and Estimation*. New York, NY: Springer-Verlag, 1994.
- [10] H. Tang, "Some physical layer issues of wide-band cognitive radio systems," in *Proc. IEEE Int. Symp. New Frontiers on Dyn. Spectr. Access (DySPAN)*, Nov. 2005, pp. 151–159.
- [11] P. Sutton, K. Nolan, and L. Doyle, "Cyclostationary signatures in practical cognitive radio applications," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 13–24, Jan. 2008.
- [12] J. G. Proakis, *Digital Communications*, 4th ed., ser. Science/Engineering/Math. McGraw-Hill, Aug. 2000.
- [13] S. M. Kay, *Fundamentals of Statistical Signal Processing*. Upper Saddle River, NJ: Prentice Hall, 1998, vol. 2 (Detection Theory).
- [14] T. J. Lim, R. Zhang, Y. C. Liang, and Y. Zeng, "GLRT-based spectrum sensing for cognitive radio," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2008, pp. 4391–4395.
- [15] R. Zhang, T. Lim, Y.-C. Liang, and Y. Zeng, "Multi-antenna based spectrum sensing for cognitive radios: A GLRT approach," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 84–88, Jan. 2010.
- [16] J. Font-Segura and X. Wang, "GLRT-based spectrum sensing for cognitive radio with prior information," *IEEE Trans. Commun.*, vol. 58, no. 7, pp. 2137–2146, Jul. 2010.
- [17] A. I. Pérez-Neira, M. A. Lagunas, M. A. Rojas, and P. Stoica, "Correlation matching approach for spectrum sensing in open spectrum communications," *IEEE Trans. Signal Process.*, vol. 57, no. 12, pp. 4823–4836, Dec. 2009.