

PERFORMANCE ANALYSIS OF SPACE-TIME BLOCK CODING WITH ADAPTIVE MODULATION

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Abstract - We present a general performance analysis of Space-Time Block Coding (STBC) with fixed and variable data rate for wireless communications. The performance is measured in terms of the tradeoff curves between diversity and spatial multiplexing gains, and in terms of symbol error rate (*SER*) for QAM constellations. Effective data rate and asymptotic approximation of *SER* expressions for fixed and adaptive modulation are found.

Keywords – Space-Time Block Coding, Diversity, Symbol Error Rate, Adaptive Modulation, Trade-off analysis.

I. INTRODUCTION

In the past few years, a significant progress in code design for transmit diversity over wireless channel has been made. In most practical systems only the receiver knows the channel. For this scenario, Space-Time Block Coding (STBC) and Space-Time Trellis Coding (STTC) have been proposed. These coding techniques permit to obtain significant gains exploiting independent available channels in a multi-input multi-output (MIMO) systems. In general, STTC perform better than STBC, but their decoding complexity tend to be much greater. On the other hand, STBC, developed on the base of a simple scheme for transmission using two transmit antennas [1,2,3], are able to achieve full diversity. Because of the orthogonal design, these coding techniques retain the property of having a very simple decoding algorithm, based only on linear processing at the receiver. In addition, STBC achieve full diversity gain but also limited spatial multiplexing gain [7]. Nevertheless, combining STBC with adaptive modulation (increasing the size of the constellation according to the mean receive signal power) data rate can be increased at the expense of loosing some diversity advantages.

This work presents a general performance analysis of STBC for slow and flat fading with fixed and variable size of symbol constellation. The performance is measured in terms of the tradeoff curves, between diversity and spatial multiplexing gains, and in terms of symbol error rate (*SER*) for QAM constellations, which are obtained from a general and compact instantaneous signal to noise power ratio expression (deduced for this work, but demonstration not included for space limitation). Effective data rate and analytical asymptotic approximations of *SER* expressions for fixed and adaptive modulation are also found.

II. CODE CONSTRUCTION OF STBC

STBC are constructed as P by N transmission matrices $\mathcal{S}_N = \{s_{p,n}\}$, whose elements are in general linear combinations of signal constellation components taken from an information block of K independent symbols [2,3]. A total of N sequences of $P \geq K$ symbols are transmitted simultaneously from each of the N antennas. The signal observed at each of the M receive antennas is a linear superposition of the N transmitted signals, perturbed by noise.

Related transmission schemes include coding matrices for 2 up to 8 transmit antennas and for an arbitrary number of receive antennas, which can make use of complex constellations to increase the spectral efficiency [2,3]. For real constellations, a symbol vector to be transmitted, $X = [x_1 \ x_2 \ \dots \ x_K]^T$, $P=K$, is coded as the matrix

$$\mathcal{S}_N = \mathbf{C}_N = \{c_{p,n}\} = \begin{bmatrix} x_1 & x_2 & \dots & x_N \\ -x_2 & x_1 & & \vdots \\ \vdots & & \ddots & \\ -x_K & \dots & & \end{bmatrix} \quad (1)$$

where $N \leq K$ and $c_{p,n} = \delta_{p,n} x_{k(p,n)}$. Columns of \mathbf{C}_N are permutations of X (according to the index $k(p,n)$) which elements are properly multiplied by $\delta_{p,n} = \pm 1$ to provide orthogonal columns.

For $N=2, 4$ and 8 , the basic square matrices are specified in [2]. For complex constellations, the coding matrix for $N=2$ is $\mathbf{G}_2 = [[x_1 \ -x_2]^T \ [x_2 \ x_1]^T]^T$. For $N=4$ and 8 , matrices are constructed from the real matrix \mathbf{C}_4 and \mathbf{C}_8 (with complex symbols), respectively, as $\mathbf{G}_N = [\mathbf{C}_N^T \ \mathbf{C}_N^*]^T$. The superscript “ T ” and “ $*$ ” mean transpose and transpose and conjugated, respectively. For real and complex constellations, and $N=3$, the matrices \mathbf{C}_3 and \mathbf{G}_3 are constructed using any 3 columns of \mathbf{C}_4 and \mathbf{G}_4 , respectively, for $N=5, 6$ and 7 , the matrices $\mathbf{C}_5, \mathbf{C}_6$ and \mathbf{C}_7 , and $\mathbf{G}_5, \mathbf{G}_6$ and \mathbf{G}_7 are constructed using any 5, 6 and 7 columns of \mathbf{C}_8 and \mathbf{G}_8 , respectively. The ratio $r_c = K/P$ can be defined as the matrix coding rate, that is the rate of independent symbols transmitted per block from each antenna, r_c is directly related with the spectral efficiency of STBC [3]. Two additional special cases for $N=4$ are the coding matrices for complex constellations, \mathbf{H}_4 [3], which is a spectral efficient matrix with $r_c=3/4$ and \mathbf{B}_4 [4], which is a

sub-optimal matrix in terms of signal to noise ratio gain, with $r_c=1$.

III. GENERAL PERFORMANCE ANALYSIS OF STBC

For flat fading assumption, the MIMO channel is represented by the matrix $\mathbf{H}(t)=\{h_{n,m}(t)\}$, where $h_{n,m}(t)$ is the complex channel gain from the transmit antenna n to the receive antenna m . For quasi-static fading assumption, i.e. the fading is static over a block of K symbols, the complex baseband receive signal matrix can be written as $\mathbf{R}=[\mathbf{R}_1 \ \mathbf{R}_2 \ \dots \ \mathbf{R}_M]=\mathcal{S}_N \mathbf{H} + \mathbf{\Xi}$, where $\mathbf{\Xi}=[\mathbf{\Xi}_1 \ \mathbf{\Xi}_2 \ \dots \ \mathbf{\Xi}_M]$ is the baseband receive noise matrix, $\mathbf{R}_m=[r_{1,m} \ r_{2,m} \ \dots \ r_{P,m}]^T$ and $\mathbf{\Xi}_m=[n_{1,m} \ n_{2,m} \ \dots \ n_{P,m}]^T$. The elements $n_{p,m}$ are assumed to be independent samples of a zero-mean complex Gaussian random variable with variance σ_n^2 . The channel gains $h_{n,m}$ are modeled to provide $E[|h_{n,m}|^2]=1$ and the total signal power σ_s^2 is divided into N , so that the expected signal to noise power ratio at each receive antenna is $SNR=\sigma_s^2/\sigma_n^2$. Assuming perfect channel state information, we have analyzed and found that in general a linear processing (L) over the receive signal matrix \mathbf{R} can be applied, to obtain the following compact and general expression for the estimation of the transmitted symbol vector

$$\hat{\mathbf{X}}=[\hat{x}_1 \ \hat{x}_2 \ \dots \ \hat{x}_K]^T = \mathbf{L} \{\mathbf{R}\} = \gamma \mathbf{X} + \mathbf{\Xi}_o \quad (2)$$

where $\gamma = \sum_{m=1}^M \sum_{n=1}^N |h_{n,m}|^2$ and $\mathbf{\Xi}_o = [\eta_1 \ \eta_2 \ \dots \ \eta_K]^T$.

so we can see that STBC can provide diversity gain of order NM for decoupled received symbols x_k corrupted by η_k , which are linear combinations of the Gaussian noise $n_{p,m}$. From this result, we have derived that the equivalent instantaneous snr for the received symbols x_k (excluding the case of matrix \mathbf{B}_4) can be expressed as

$$snr = \frac{\sigma_s^2}{\sigma_n^2} = \gamma \frac{SNR}{N \cdot r_c} \quad (3)$$

In a previous work [5], a particular case of this expression is obtained, but only from simulations.

The linear processing to be applied over \mathbf{R} depends on the coding matrix (\mathbf{G}_2 , \mathbf{B}_4 , $\mathbf{C}_{4,8}$, $\mathbf{G}_{4,8}$ or \mathbf{H}_4 , see the references to find the specific coding matrices for each case) as summarized in table 1, where the considered matrices are the following.

i) for matrix \mathbf{G}_2 (real and complex constellations, $N=K=P=2$, $r_c=1$)

$$\mathbf{A}_m = \begin{bmatrix} h_{1,m} & h_{2,m} \\ h_{2,m}^* & -h_{1,m}^* \end{bmatrix} \quad \text{and} \quad \mathbf{R}'_m = \begin{bmatrix} r_{1,m} \\ r_{2,m}^* \end{bmatrix}$$

ii) for matrix \mathbf{C}_4 (real constellations, $N=K=P=4$, $r_c=1$)

$$\mathbf{A}_m = \begin{bmatrix} h_{1,m} & h_{2,m} & h_{3,m} & h_{4,m} \\ h_{2,m} & -h_{1,m} & h_{4,m} & -h_{3,m} \\ h_{3,m} & -h_{4,m} & -h_{1,m} & h_{2,m} \\ h_{4,m} & h_{3,m} & -h_{2,m} & -h_{1,m} \end{bmatrix}$$

Extension of this result can be done for \mathbf{C}_8 (real constellations, $N=K=P=8$, $r_c=1$)

iii) for matrix \mathbf{G}_4 (complex constellations, $N=K=4$, $P=8$, $r_c=1/2$). \mathbf{A}_m is the same matrix that in the previous case *ii*,

$\mathbf{R}1_m = [r_{1,m} \ r_{2,m} \ r_{3,m} \ r_{4,m}]^T$ and $\mathbf{R}2_m = [r_{5,m} \ r_{6,m} \ r_{7,m} \ r_{8,m}]^T$

Extension of this result can be done for \mathbf{G}_8 (complex constellations, $N=K=8$, $P=16$, $r_c=1/2$)

iv) for matrix \mathbf{H}_4 , (complex constellations, $N=P=4$, $K=3$, $r_c=3/4$)

$$\mathbf{A}_m = \begin{bmatrix} h_{1,m} & h_{2,m} & \frac{h_{3,m} + h_{4,m}}{\sqrt{2}} \\ 0 & 0 & \frac{h_{3,m} - h_{4,m}}{\sqrt{2}} \\ \frac{h_{4,m} - h_{3,m}}{2} & \frac{h_{3,m} - h_{4,m}}{2} & 0 \\ \frac{h_{3,m} - h_{4,m}}{2} & \frac{h_{3,m} - h_{4,m}}{2} & 0 \end{bmatrix}$$

$$\text{and} \quad \mathbf{D}_m = \begin{bmatrix} 0 & 0 & 0 \\ h_{2,m} & -h_{1,m} & 0 \\ \frac{-h_{4,m} - h_{3,m}}{2} & \frac{-h_{3,m} - h_{4,m}}{2} & \frac{h_{1,m} + h_{2,m}}{\sqrt{2}} \\ \frac{-h_{3,m} - h_{4,m}}{2} & \frac{h_{3,m} + h_{4,m}}{2} & \frac{h_{1,m} - h_{2,m}}{\sqrt{2}} \end{bmatrix}$$

v) for the special (sub-optimal) case of matrix \mathbf{B}_4 (complex constellations, $N=K=P=4$, $r_c=1$)

$$\mathbf{A}_m = \frac{\gamma_m}{\gamma_m^2 + \alpha_m^2} \left(\alpha_m \begin{bmatrix} h_3 & -h_4 & -h_1 & h_2 \\ -h_4^* & -h_3^* & h_2^* & h_1^* \\ h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \end{bmatrix} + \gamma_m \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3 & h_4 & h_1 & -h_2 \\ -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix} \right)$$

where $\gamma_m = \sum_{n=1}^N |h_{n,m}|^2$ and $\alpha_m = 2j \text{Im}\{h_{1,m}^* h_{3,m} + h_{4,m}^* h_{2,m}\}$

and $\mathbf{R}'_m = [r_{1,m} \ r_{2,m}^* \ r_{3,m} \ r_{4,m}^*]^T$

In this case, the snr for the received symbols x_k is

$$snr = \frac{\sigma_s^2}{\sigma_n^2} = \gamma^2 \left(\sum_{m=1}^M \gamma_m^2 + \alpha_m^2 \right)^{-1} \frac{SNR}{4} \quad (4)$$

Linear processing as stated in *ii*, *iii* and *v* can be also applied when matrices \mathbf{C}_4 , \mathbf{G}_4 or \mathbf{H}_4 are used for $N=3$ transmit antennas (and \mathbf{C}_8 or \mathbf{G}_8 for $N=5, 6$ or 7) setting to zero the proper $h_{n,m}$.

Table 1
Linear processing A for the estimation of \hat{X}

G_2, B_4	$C_{4,8}$	$G_{4,8}$	H_4
$\sum_{m=1}^M A_m^t R_m^*$	$\text{Re}\left\{\sum_{m=1}^M A_m^t R_m\right\}$	$\frac{1}{2}\sum_{m=1}^M A_m^t R_{I_m} + A_m^t R_{2_m}^*$	$\sum_{m=1}^M A_m^t R_m + D_m^t R_m^*$

IV. DIVERSITY AND SPATIAL MULTIPLEXING TRADEOFF

Following the pioneering work of [7], a MIMO channel of NM antennas can provide two extreme points of maximal diversity gain NM (full diversity) or maximal spatial multiplexing gain $\min\{N, M\}$. This means that the maximal SNR exponent of the error rate and the maximal number of parallel channels provided by the MIMO channel in order to transmit independent information are obtained, both at high SNR. Given a MIMO channel, both gains can be simultaneously obtained, but there is a fundamental tradeoff between how much of each type of gain can be extracted. In [7] it is established that the limit of the tradeoff curve between diversity gain (d) and spatial multiplexing gain (r) that any scheme can achieve in the Rayleigh-fading multiple-antenna channel, is a optimal tradeoff curve $d^*(r)$ given by the piecewise-linear function connecting the points $(k, d^*(r))$, $k=0, 1, \dots, \min\{N, M\}$, where $d^*(k)=(N-k)(M-k)$. The tradeoff and optimal tradeoff curves bridge the gap between the two design criteria, that is, diversity and spatial multiplexing gains, however, increasing the diversity advantage comes at a price of decreasing the spatial multiplexing gain, and vice versa.

In addition to providing diversity to improve reliability, multiple-antenna channels can also support higher data rate than single-antenna systems exploiting the spatial parallel channels [7]. Therefore, since the channel capacity increases linearly with $\log SNR$, in order to achieve a certain fraction of the capacity at high SNR, we consider schemes that support a data rate which also increases with $\log SNR$. Let $M_C = aSNR^r$ be the size of the constellation which supports $\log_2 M_C$ [b/symbol], with a an arbitrary constant, then the data rate is $R(SNR) = \log_2 a + r \log_2 SNR$ [b/s/Hz], and thus the spatial multiplexing gain achieved by this scheme at high SNR is r . If STBC is used, the effective data rate depends directly of the matrix coding rate r_c , that is $R(SNR) = r_c \log_2 a + r_c r \log_2 SNR$ [b/s/Hz], and the effective spatial multiplexing gain is $r_c r$.

V. ERROR RATE ANALYSIS

We start deriving the asymptotic symbol error rate of an arbitrary uncoded square M_C -QAM constellation for a scheme with one transmit antenna and $L=NM$ receive antennas, using the fact that, from the diversity point of view, STBC are equivalent to a maximal ratio receive combiner (MRRC). Next we extend the results to an

adaptive size constellation ($0 < r < 1$) combined with STBC (similar analysis can also be obtained for PSK and non-square QAM constellations but is not included here because of lack of space).

The instantaneous error probability of a M_C -QAM symbol is $p_{es} = 1 - (1 - p_{\sqrt{M_C}})^2$, where $p_{\sqrt{M_C}}$ is the instantaneous error probability of the independent and orthogonal $\sqrt{M_C}$ -PAM signals that generate the M_C -QAM constellation. Since the noise is Gaussian, the $\sqrt{M_C}$ -PAM instantaneous error probability is given by

$$p_{\sqrt{M_C}} = \left(1 - \frac{1}{\sqrt{M_C}}\right) \text{erfc}\left(\sqrt{\frac{3/2}{M_C - 1} snr}\right) \quad (5)$$

where $snr = \rho \cdot \gamma$ is the instantaneous signal-noise ratio, ρ is the mean signal-noise ratio received per antenna, and $\gamma = \sum_{k=1}^L |h_k|^2$. The complex coefficients h_k are normalized so that $E[|h_k|^2] = 1$.

The symbol error rate is

$$SER = E[p_{es}] = E\left[1 - (1 - p_{\sqrt{M_C}})^2\right] \quad (6)$$

which, for $\rho \rightarrow \infty$ ($p_{\sqrt{M_C}} \rightarrow 0$), (6) can be approximated by

$$SER \approx 2E[p_{\sqrt{M_C}}] = 2\left(1 - \frac{1}{\sqrt{M_C}}\right) E\left[\text{erfc}\left(\sqrt{SNR' \gamma}\right)\right] \quad (7)$$

where $SNR' = \frac{3/2}{M_C - 1} \rho$

For complex Gaussian distribution of the coefficients h_k (Rayleigh channel, the typical scenario for practical wireless applications), there is a closed-form solution for (7) [6], which can be expressed as

$$SER \approx 4\left(1 - \frac{1}{\sqrt{M_C}}\right) \left[\frac{1}{2}(1 - \mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1 + \mu)\right]^k \quad (8)$$

where $\mu = \sqrt{\frac{SNR'}{1 + SNR'}}$

again, for $SNR' \rightarrow \infty$ ($\mu \rightarrow 1$), (8) can be approximated by

$$SER \approx 4\left(1 - \frac{1}{\sqrt{M_C}}\right) \binom{2L-1}{L} \frac{1}{(4SNR')^L} \quad (9)$$

then

$$SER \approx 4\left(1 - \frac{1}{\sqrt{M_C}}\right) \binom{2L-1}{L} \left(\frac{M_C - 1}{6}\right)^L \rho^{-L} \quad (10)$$

Including now a family of adaptive M_C -QAM constellations, following size $M_C = aSNR^r$, and STBC where $\rho = SNR/N/r_c$, $L = NM$, the expression (10) at high SNR can be simplified to

$$SER \approx \alpha SNR^{-NM(1-r)} \quad (11)$$

$$\text{where } \alpha = 4 \left(\frac{ar_c N}{6} \right)^{NM} \binom{2NM-1}{NM}$$

It can be shown that (11) is an asymptotic approximation for the SER at high SNR but also an upper bound, so that $SER = \alpha SNR^{-NM(1-r)} - \beta$, where $\beta > 0$ and $\beta \rightarrow 0$ when $SNR \rightarrow \infty$, and therefore the approximation (11) can also be used to determinate the minimum SNR that provides a certain maximum absolute error when using the approximation.

For the special case of BPSK constellation, there is a closed-form solution for the bit error rate, which can be expressed as

$$BER = \left[\frac{1}{2}(1-\mu) \right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu) \right]^k \quad (12)$$

$$\text{where } \mu = \sqrt{\frac{SNR}{r_c N + SNR}}$$

At high SNR, expression (12) can also be asymptotically approximated by

$$BER = \left(\frac{r_c N}{4} \right)^L \binom{2NM-1}{NM} SNR^{-NM} \quad (13)$$

From the asymptotical results for the SER, it can be concluded that STBC provide full diversity gain of NM for fixed data rate ($r=0$) and diversity gain of $NM(1-r)$ for adaptive modulation with $0 < r < 1$.

Combining the result for the spatial multiplexing gain obtained in section IV and the diversity gain, we can compare the optimal tradeoff curves for a $N \times M$ MIMO channel and for some $N \times M$ STBC examples, as shown in Fig. 1. As it can be seen from the figure, only the scheme of coding matrix G_2 with $M=1$ can achieve the optimal diversity gain for any spatial multiplexing gain, for the rest of the schemes the spatial multiplexing gain is at most the matrix coding rate. Nevertheless for all cases the maximal diversity gain is optimal for fixed data rate ($r=0$). The limit of spatial multiplexing of orthogonal design is $r=1$, since full rate essentially means that one symbol is transmitted per symbol time.

Figures 2, 3 and 4 show SER vs. SNR performance for some STBC schemes, SER approximation (expression 8) and asymptotic SER (expression 10) for fixed ($r=0$) and asymptotic SER (expression 11) for variable data rate ($r > 0$). In figures 2, 3 and 4, the asymptotic SER for variable data rate have been calculated for the parameter $r=0.5, 0.7$ and 0.9 , and for the constant $\alpha=0.5, 0.25$ and 0.5 , respectively. Fig. 2 shows SER for schemes of coding matrices with coding rate $1/2$, G_2 with $M=2$ and G_4 with $M=1$, which have

exactly the same performance because of the same products $L=NM$ and $r_c N$. Fig. 3 shows SER for the scheme of coding matrix with coding rate $1/4$, H_4 with $M=1$. And Fig. 4 shows SER for a scheme with more diversity gain of matrix with coding rate $1/2$, G_4 with $M=2$.

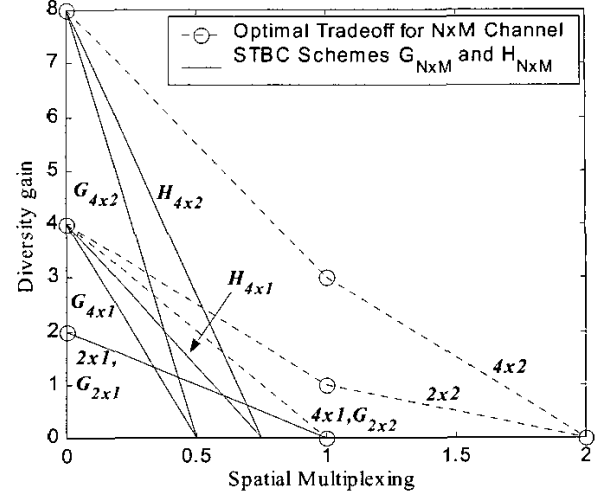


Fig. 1. Diversity-multiplexing tradeoff for some STBC with adaptive modulation.

From the results of figures 2, 3 and 4, we can conclude that, in general, the asymptotic approximation of SER is very good for high SNR, although, for higher diversity gains, the error of the asymptotic approximation for low SNR is more significant.

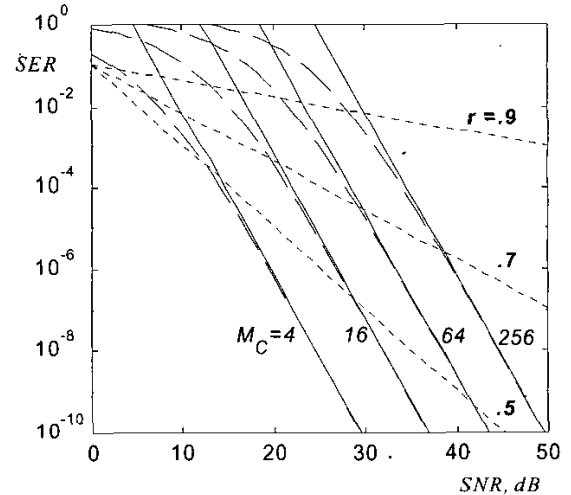


Fig. 2. SER vs. SNR performance of STBC with adaptive modulation for coding matrices G_2 with $M=2$ and G_4 with $M=1$. SER approximation (8) and asymptotic SER (10) for fixed data rate (dashed and solid lines, respectively) and asymptotic SER (11) for variable data rate (dotted line).

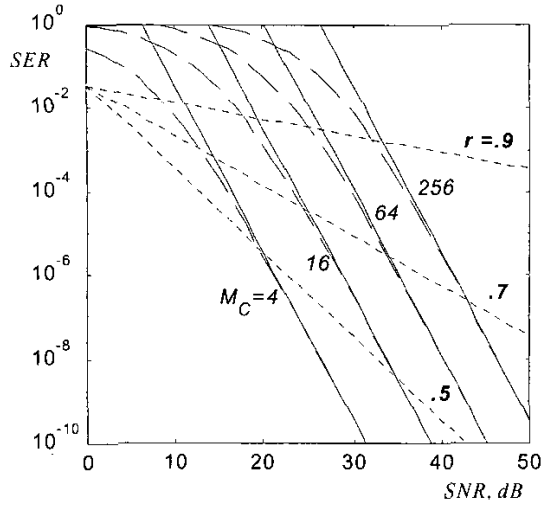


Fig. 3. *SER* vs. *SNR* performance of STBC with adaptive modulation for coding matrix H_4 with $M=1$. *SER* approximation (8) and asymptotic *SER* (10) for fixed data rate (dashed and solid lines, respectively) and asymptotic *SER* (11) for variable data rate (dotted line).

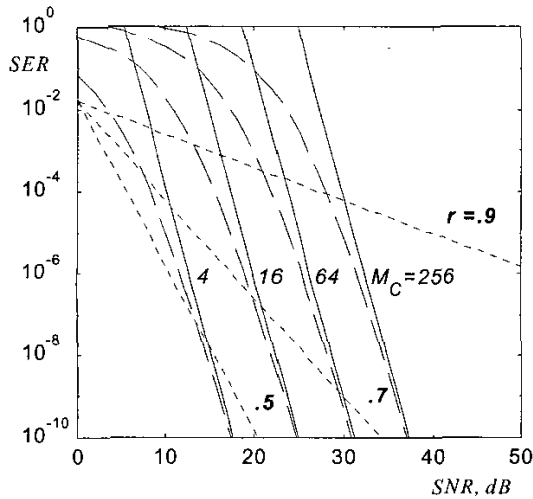


Fig. 4. *SER* vs. *SNR* performance of STBC with adaptive modulation for coding matrix G_4 with $M=2$. *SER* approximation (8) and asymptotic *SER* (10) for fixed data rate (dashed and solid lines, respectively) and asymptotic *SER* (11) for variable data rate (dotted line).

It can also be concluded from figures 2, 3 and 4, that despite the spatial multiplexing gain of STBC is limited to the matrix coding rate, the diversity gain can increase with the product NM .

VI. CONCLUSIONS

A general performance analysis of STBC has been presented for slow flat fading with fixed and variable size of constellations to support variable data rate. The performance has been evaluated analyzing the tradeoff curves between diversity and spatial multiplexing gain and in terms of the symbol error rate for square QAM constellations. Effective data rate and analytical asymptotic approximation for high *SNR* of *SER* expressions for fixed and adaptive modulation have been derived and evaluated.

As expected STBC achieve full diversity gain but also limited spatial multiplexing gain, however, combining STBC with adaptive modulation by increasing the size of the constellation according to *SNR*, it is possible to increase data rate at the expense of diversity advantages. Increasing de size of the constellation, the data rate increases, but also the *SER* exponent decreases. Nevertheless, limiting the parameter r to $0 < r < 1$, reliable communication is possible. On the other side, spatial multiplexing and diversity gains can increase by increasing N and/or M .

Finally, in any real system, it should be considered that in order to increase the data rate according $\log SNR$, the size of the constellation can only be integer, so that the asymptotic *SER* approximation shows the *SER* tendency of an adaptive QAM modulation by segments of *SNR*.

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